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B.A./B.Sc. (General) 3rd Semester

(2123)

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MATHEMATICS

Paper : I (Advanced Calculus-I)

Time Allowed : Three Hours] [Maximum Marks : 30

Note :— Attempt **five** questions in all selecting at least **two** questions from each unit. All questions carry equal marks.

UNIT–I

1. (a) Prove that $\lim\limits_{(x,y)\to(0,0)}\left(x\sin\frac{1}{y}+\frac{xy}{\sqrt{x^2+y^2}}\right)=0$

(b) Discuss the continuity of the function :

$$f(x,y)=\begin{cases}\dfrac{x^2y}{x^4+y^2} & , \quad (x,y)\neq(0,0)\\ 0 & , \quad (x,y)=(0,0)\end{cases}$$

at the point (0,0).

2. (a) If $u=\log(x^3+y^3+z^3-3xyz)$ then show that :

$$\left(\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}+\frac{\partial u}{\partial z}\right)^2=\frac{9}{(x+y+z)^2}.$$

(b) If $x^y + y^x = (x + y)^{x+y}$ then by using partial derivatives find $\frac{dy}{dx}$.

3. (a) Let $f(x,y) = \begin{cases} \frac{xy}{x^2+y^2} & , (x,y) \neq (0,0) \\ 0 & , (x,y) = (0,0) \end{cases}$.

Prove that $f_x(0,0)$ and $f_y(0,0)$ both exist but f is not differentiable at (0,0).

(b) Show that f (x,y) = cos (x + y) is differentiable at $\left(\frac{\pi}{4}, \frac{\pi}{4}\right)$.

4. (a) Find directional derivative of $x^2yz + 4xz^2$ at (1, –2, –1) in the direction of $2\hat{i} - \hat{j} - 2\hat{k}$.

(b) If $\vec{a}$ is a Constant Vector, then show that :

$$\nabla \times \left(\frac{\vec{a} \times \vec{r}}{r^3}\right) = -\frac{\vec{a}}{r^3} + \frac{3(\vec{a} \cdot \vec{r})}{r^5}\vec{r}.$$

UNIT–II

5. (a) State and prove Euler's theorem on homogenous functions of two variables.

(b) Use Taylor's theorem to expand $xy^2 + 3x - 2$ in power of (x + 2) and (y – 1).

6. (a) If x + y + z = u, y + z = uv, z = uvw then show that

$$\frac{\partial(x,y,z)}{\partial(u,v,w)} = u^2v.$$

(b) Show that $f_1(x, y) = \frac{x + y}{1 - xy}$ and $f_2(x, y) = \tan^{-1} x + \tan^{-1} y$ are functionally dependent.

7. (a) Find the envelope of a system of concentric and co-axial ellipses of constant area.

(b) Find the evolute of the Parabola $y^2 = 4ax$ regarding it as an envelope of its normals.

8. (a) Find the extreme values of the function :

$f(x, y) = (x - y)^4 + (y - 1)^4$.

(b) Show that the volume of the largest Parallelopiped that can be inscribed in the sphere $x^2 + y^2 + z^2 = a^2$ is $\frac{8a^3}{3\sqrt{3}}$.