(i) **Printed Pages : 3** Roll No. ....

(ii) **Questions : 8**

Sub. Code : | 0 | 4 | 4 | 3 |

Exam. Code : | 0 | 0 | 0 | 5 |

B.A./B.Sc. (General) 5th Semester

(2123)

MATHEMATICS

Paper—I : Analysis—I

Time Allowed : Three Hours] **[Maximum Marks : 30**

Note :—Attempt **five** questions in all selecting at least **two** questions from each unit.

UNIT–I

1. (a) Define equivalent sets. Prove that open intervals (3,6) and (6,9) are equivalent sets.

   (b) Evaluate L (P,f) where $f(x) = x^2$ and $P = \left\{-\frac{3}{2}, -\frac{1}{2}, \frac{1}{4}, 1\right\}$.

   3+3=6

2. (a) Prove that every monotonically decreasing function defined on closed interval [5, 7] is Riemann integrable.

   (b) State and prove fundamental theorem of integral calculus.

   3+3=6

3. (a) Prove that $\frac{\sqrt{3}}{8} \le \int_{\pi/4}^{\pi/3} \frac{\sin x}{x} dx \le \frac{\sqrt{2}}{6}$.

(b) If $B(m, n) = \int_{-1}^{\infty} \frac{x+1}{(x+2)^6} dx$ then find "m" and "n". Also evaluate B (m, n). 3+3=6

4. (a) State Duplication formula. Use it to show that :

$$\int_0^{\infty} \frac{1}{1+y^4} dy = \frac{\pi\sqrt{2}}{4}$$

(b) Express the integral as Beta function $\int_0^3 x^3 (27-x^3)^{-1/3} dx$.

3+3=6

UNIT–II

5. (a) Show that the improper integral $\int_a^{\infty} \frac{dx}{x^p} (a > 0)$ is convergent at ∞, if $p \leq 1$ and divergent at ∞ if, $p \leq 1$.

(b) Use Dirichlet's test to show that $\int_0^{\infty} \frac{\sin x}{x} dx$ is convergent at ∞. 3+3=6

6. (a) Discuss the convergence of the integral $\int_1^2 \frac{dx}{(x-1)^{1/2} (2-x)^{1/3}}$.

(b) State and prove Abel's test for convergence of improper integral. 3+3=6

7. (a) Discuss the convergence of

$$\int_0^1 x^{n-1} \log x \, dx \cdot$$

(b) Use Frullani's Integral to prove

$$\int_0^\infty \frac{\tan^{-1}(ax) - \tan^{-1}(bx)}{x} dx = \frac{\pi}{2} \log\left(\frac{a}{b}\right)$$ 3+3=6

8. (a) Show that $\int_0^\infty \frac{\tan^{-1}(ax)}{x(1+x^2)} dx = \frac{\pi}{2} \log(1+a); a \geq 0$.

(b) Evaluate $\int_0^\infty e^{-\alpha x} \frac{\sin \beta x}{x} dx$ for $\alpha \geq 0$. Hence show that

$$\int_0^\infty \frac{\sin \beta x}{x} dx = \pi/2 .$$ 3+3=6