(i) Printed Pages : 2 Roll No. ....

(ii) Questions : 8

Sub. Code : | 0 | 4 | 4 | 4 |

Exam. Code : | 0 | 0 | 0 | 5 |

B.A./B.Sc. (General) 5th Semester

(2123)

MATHEMATICS

Paper—II : Modern Algebra

Time Allowed : Three Hours] **[Maximum Marks : 30**

Note :— Attempt **FIVE** questions in all by selecting at least **TWO** from each unit.

UNIT—I

1. (a) Prove that if G is semi group in which both cancellation Laws hold is a group, where G is finite. 3

   (b) Show that Abelian group of order 6 is cyclic. 3

2. (a) State and prove Lagrange's theorem on groups. 3

   (b) Prove that subgroup of index 2 is normal in that group. 3

3. (a) Show that every group of order p^2 where p is prime number is Abelian. 3

   (b) State and prove Cayley's theorem. 3

4. (a) Show that subgroup of cyclic group is cyclic. 3

   (b) Find cosets of 2Z in Z. 3

UNIT—II

5. (a) If I & J are ideals of ring R then show that $I \cap J$ is also ideal of R. 3

   (b) Show that there does not exist any I.D. with 10 elements; where I.D. is integral domain. 3

6. (a) Define maximal and prime ideals of a ring and show that 4Z is maximal ideal of 2Z. 3

   (b) If in a ring R, $x^3 = x \ \forall \ x \in R$, then show that R is commutative. 3

7. (a) If $f : R \rightarrow R'$ is ring homomorphism then show that f is 1 – 1 iff $\ker f = \{ 0 \}$. 3

   (b) Find all units of $Z_5[x]$. 3

8. Whow that M is maximal ideal of R iff R/m is field. 6