UUD-31 Date - 27/12/2023(M)

(i) Printed Pages : 3 Roll No. ....

(ii) Questions : 8

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B.A./B.Sc. (General) 5th Semester

(2123)

MATHEMATICS

Paper—III : Probability Theory

Time Allowed : Three Hours] **[Maximum Marks : 30**

Note :— Attempt **five** questions in all selecting at least two questions from each unit. All questions carry equal marks.

UNIT–I

1. (a) Prove that, for any n events $A_1, A_2 \dots A_n$,

$$P\left(\bigcup_{i=1}^{n} Ai\right) \le \sum_{i=1}^{n} P(A_i)$$

(b) State and prove Total Probability Theorem. 3,3

2. (a) Find constant c so that $f(x)=\begin{cases} cx(3-x)^4 & , 0<x<3 \\ 0 & , \text{elsewhere} \end{cases}$

is a p.d.f.

(b) The probability density function of a random variable X is

given as $f(x)=\begin{cases} 4x(1-x)^2 & , \quad 0 \le x \le 1 \\ 0 & , \quad \text{otherwise} \end{cases}$.

Evaluate (i) Mean (ii) Median (iii) Mode of x. 3,3

3. (a) For Poisson random variable X, $E(X^2) = 20$ find E (X).

(b) A die is thrown 6 times. If getting an odd number is a success, what are the chances of having (i) 5 successes (ii) at least 5 successes (iii) at most 5 successes. 3,3

4. Show that Poisson Distribution is limiting case of Binomial Distribution. 6

UNIT–II

5. (a) If X is uniformly distributed on (0, 30). Find its cumulative distribution function. P.d.f. of X is

$$f(x)=\begin{cases} \frac{1}{30-0}, & 0 \leq x \leq 30 \\ 0, & \text{otherwise} \end{cases}$$

(b) A random variable X has exponential distribution with parameter $\lambda = 3$ find :

(i) $P(X \geq 4)$

(ii) Find S.D and coefficient of variation. 3,3

6. (a) Find Mgf of normal distribution.

(b) In normal distribution, 7% items are below 35 and 89% are below 63. Find mean and S.D. of distribution. 3,3

7. Let X and Y have a joint prob. distribution function as :

$$f(x,y)=\begin{cases} 6x^2y & , \quad 0<x<1, 0<y<1 \\ 0 & , \quad \text{elsewhere} \end{cases}$$

(a) Find P (X + Y < 1)

(b) Find P (X > Y)

(c) Find P (X < 1/ Y < 2)

(d) Find $P\left(0 < X < \frac{3}{4}, \frac{1}{3} < Y < 2\right)$ 6

8. Prove that coefficient of correlation is independent of change of origin and scale. 6