(i) Printed Pages: 3 Roll No. .....

(ii) Questions : 8 Sub. Code : 0145

Exam. Code : 0002

B.A./B.Sc. (General) 2nd Semester

(2054)

MATHEMATICS

Paper—I (Solid Geometry)

Time Allowed : Three Hours] **[Maximum Marks : 30**

Note :—Attempt any **FIVE** questions in all, selecting at least **TWO** questions from each Unit. All questions carry equal marks.

UNIT—I

1. (a) Shift the origin to a suitable point so that the equation $2x^2 - 2y^2 + z^2 - 4x + 8y + 2z - 5 = 0$ is transformed into an equation in which the first degree terms are absent.

   (b) Find the transformed equation of the surface

   $$x^2 + 3y^2 + 5z^2 + 2yz + 2zx + 2xy = 1$$

   referred to axes through the same origin and having direction numbers $<2, 2, 1>$, $<-2, 1, 2>$, $<1, -2, 2>$.

2. (a) Find the equations to spheres which pass through the circle $x^2 + y^2 + z^2 = 5$, $x + 2y + 3z = 3$ and touch the plane $4x + 3y = 15$.

(b) Find the equation of the sphere through the circle

$$x^2 + y^2 + z^2 - 2x + 3y - 4z + 6 = 0,$$

$$3x - 4y + 5z - 15 = 0$$

and cutting the sphere $x^2 + y^2 + z^2 + 2x + 4y - 6z + 11 = 0$ orthogonally.

3. (a) Find the limiting points of co-axial system defined by the spheres

$$x^2 + y^2 + z^2 + 3x - 3y + 6 = 0,$$

$$x^2 + y^2 + z^2 - 6y - 6z + 6 = 0.$$

(b) Find the equation of the enveloping cylinder of the sphere $x^2 + y^2 + z^2 + 2x + 2y + 2z + 2 = 0$ and whose generators are parallel to the line $\frac{x}{1} = \frac{y}{-1} = \frac{z}{1}$.

4. (a) Find the equation of the right circular cylinder of radius 2, whose axis is the line $\frac{x-1}{2} = \frac{y-2}{1} = \frac{z-3}{2}$.

(b) Find the equation to the cylinder whose generators are parallel to $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$ and guiding curve is $x^2 + y^2 = 16$, $z = 0$.

UNIT—II

5. (a) Find the equation of the right circular cone whose vertex is at the origin, whose axis is the line $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$ and which has a semi vertical angle of 30°.

(b) Find the condition that the plane $lx + my + nz = 0$ may touch the cone $2x^2 - 3y^2 + z^2 = 0$ and find the equation of the reciprocal cone.

6. (a) Find the equation of the line in which the plane $x - 2y - z = 0$ cuts the cone $3x^2 + 4y^2 - z^2 = 0$. Also find the angle between them.

(b) If $\frac{x}{5} = \frac{y}{-4} = \frac{z}{1}$ is one of a set of three mutually perpendicular generators of the cone $5yz - 8zx - 3xy = 0$, find the equations of the other two.

7. (a) Identify the surface : $16z^2 - 4x^2 - 8z + 8x - 3 = 0$.

(b) Prove that the equation

$$4x^2 - y^2 + 2z^2 + 2xy - 3yz + 12x - 11y + 6z + 4 = 0$$

represent a cone whose vertex is $(-1, -2, -3)$.

8. Reduce the equation

$$11x^2 + 10y^2 + 6z^2 - 8yz + 4zx - 12xy + 72x - 72y + 36z + 150 = 0$$

to the standard form and show that it represent an ellipsoid, and find the equations of the axes.