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B.A./B.Sc. (General) 4th Semester

(2054)

MATHEMATICS

Paper : I (Advanced Calculus-II)

Time Allowed : Three Hours] **[Maximum Marks : 30**

Note :— Attempt **five** questions in all, selecting at least **two** questions from each unit. All questions carry equal marks.

UNIT—I

1. (a) Show that the sequence $\left\{\frac{2n+7}{3n+8}\right\}$ is convergent.

   (b) Show that the sequence $\{a_n\}$ where

$$a_n = \frac{1}{\sqrt{n^2+1}} + \frac{1}{\sqrt{n^2+2}} + \ldots\ldots + \frac{1}{\sqrt{n^2+n}}$$ converges to 1.

2. (a) State and prove Cauchy's First Theorem on limits.

   (b) Prove that a sequence is convergent iff it is a Cauchy sequence.

3. (a) If s_1 and s_2 are positive and $s_{n+1} = \sqrt{s_n s_{n-1}}$, prove that sequences $s_1, s_3, s_5, \ldots\ldots\ldots ; s_2, s_4, s_6, \ldots\ldots\ldots$ are the one increasing and the other decreasing and their common limit is $(s_1 s_2^2)^{1/3}$.

(b) Show that the sequence $\{a_n\}$ where $a_1 = 1$, $a_{n+1} = \sqrt{6 + a_n}$ converges to 3.

4. (a) Prove that the function defined by $f(x) = \sin\frac{1}{x}$, $x \in \mathbb{R}^+$ is continuous but not uniformly continuous on $\mathbb{R}^+$.

(b) Using concept of sequential continuity, show that the function

$$f(x) = \begin{cases} 1 & \text{if } x \text{ is rational} \\ -1 & \text{if } x \text{ is irrational} \end{cases}$$

is discontinuous at every point of $\mathbb{R}$.

UNIT—II

5. (a) Show that p-series $\sum_{n=1}^{\infty} \frac{1}{n^p}$ converges if $p > 1$ and diverges if $0 < p \leq 1$.

(b) Examine the convergence or divergence of the series

$$\sum_{n=1}^{\infty} \frac{(n - \log n)^n}{n^n . 2^n}.$$

6. (a) Discuss the convergence or divergence of the series

$$1 + \frac{2x}{2!} + \frac{3^2 x^2}{3!} + \frac{4^3 x^3}{4!} + \ldots\ldots\ldots, \; x > 0.$$

(b) Discuss the covergence or divergence of the series $\frac{1^2}{2^2}+\frac{1^2.3^2}{2^2.4^2}x+\frac{1^2.3^2.5^2}{2^2.4^2.6^2}x^2+\ldots\ldots,\ x>0.$

7. (a) Test for the convergence or diverigence of the series $\sum_{n=2}^{\infty}\frac{1}{n(\log n)^p},\ p>0.$

(b) Show that series $\sum_{n=1}^{\infty}\frac{(-1)^{n-1}x^{2n-1}}{\lfloor 2n-1}$ is convergent by showing that it is absolutely convergent.

8. (a) Show that the series $x-\frac{x^2}{2}+\frac{x^3}{3}-\frac{x^4}{4}+\ldots\ldots$ is convergent for $-1<x\le 1$.

(b) Rearrange the series $1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+\frac{1}{5}\ldots\ldots$ so that its sum is $\frac{3}{2}\log 2$.