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B.A./B.Sc. (General) 4th Semester

(2054)

MATHEMATICS

Paper—III (Dynamics)

Time Allowed : Three Hours] [Maximum Marks : 30

Note :—Attempt any **FIVE** questions in all, selecting at least **TWO** questions from each Unit. All questions carry equal marks.

UNIT—I

1. (a) A point moving with uniform acceleration in a straight line describes successive equal distances in time t_1, t_2, t_3; show that :

$$\frac{1}{t_1} - \frac{1}{t_2} + \frac{1}{t_3} = \frac{3}{t_1 + t_2 + t_3}$$

(b) O, A, B, C, D are five points in a straight line such that OA = AB = BC = CD. If a particle starts from rest with constant acceleration from O, prove that the times of describing AB, BC, CD are

$$(\sqrt{2} - 1) : (\sqrt{3} - \sqrt{2}) : (\sqrt{4} - \sqrt{3})$$

2. (a) Two masses m_1, m_2 are connected by an inelastic string; m_2 is placed on a smooth horizontal table and the string passes over a light smooth pulley at the edge of the table and m_1 is hanging freely. Determine the motion and the tension in the string. Find also the pressure on the pulley.

(b) A body is projected up a smooth inclined plane of length 20 meters and inclination 30° with a velocity just sufficient for it to reach the top. Divide the whole length into three parts, so that each part is covered in the same time.

3. (a) A particle moves in a straight line, starting from rest from a distance c to a centre of attraction towards which the force per unit mass is $\frac{\mu}{x^3}$, where x is measured from the centre. Show that the time required to reach the centre is $\frac{c^2}{\sqrt{\mu}}$.

(b) A particle moves in a straight line with a retardation $\frac{k}{x^2}$, $x \neq 0$, where x is its displacement with $k > 0$. It is also known that at $t = 0$, $x = R$ and velocity $v = v_0$, and $v \to 0$ as $\to \infty$. Find the displacement of the particle at time t.

4. (a) A point moving in a straight line with S.H.M. has velocities u and v, when its distances from the centre are a and b, prove that period of motion is $2\pi\sqrt{\frac{a^2 - b^2}{v^2 - u^2}}$.

(b) A particle moving in a straight line OAB with S.H.M. has its velocity zero when it is at the points A and B, and it has a velocity v when it is half way between them. If the distances of A and B from origin O and a and b respectively, show that the periodic time is $\frac{\pi(b - a)}{v}$.

UNIT—II

5. (a) Find the expressions for velocity and acceleration of a particle moving in a plane in rectangular co-ordinate system.

(b) If the greatest height of a projectile above a horizontal plane through the point of projection be a and θ be the angle of projection, find the time between the instants at which the height of projectile is $a \sin^2\theta$.

6. (a) A particle moves along a plane curve such that its tangential and normal acceleration are equal and the angular velocity is constant. Find the path of the particle.

(b) A particle is projected along a smooth curve, from any point in it with velocity v_1, prove that v_2, the velocity of the particle, after it has moved through a height h, is given by $v_2^2 = v_1^2 - 2gh$.

7. (a) Prove that the work done in stretching an elastic string is equal to the product of the extension and mean of the initial and final tensions.

(b) A horse draws a train car of 2 tons weight along a horizontal road with a uniform velocity of $\frac{15}{2}$ miles per hour. Supposing the resistance to be $\frac{1}{80}$ of the weight. Find the rate at which he is doing work.

8. (a) Two cars P and Q are moving at 50 km/hr and 30 km/hr respectively on a road in the same direction. What is the relative velocity of Q w.r.t. P ? Also obtain this value when the cars are moving in opposite directions.

(b) A bullet of mass m kg is fired into a fixed target of mass M kg and penetrates through a distance a meters. If the target was free to move, show that the distance penetrated would be $\frac{Ma}{M + m}$ meters and that the K.E. lost would be $\frac{M}{M + m}$ of its initial distance.