Printed Pages: 3 Roll No. ....

(ii) Questions : 8

Sub. Code : 0 5 4 1

Exam. Code : 0 0 0 6

B.A./B.Sc. (General) 6th Semester

(2054)

MATHEMATICS

Paper—I : Analysis–II

Time Allowed : Three Hours] **[Maximum Marks : 30**

Note :— Attempt **FIVE** questions in all, selecting at least **TWO** questions from each Unit.

UNIT—I

1. (a) If $A = \{(x, y) \mid -2 \le x \le 0, -2 \le y \le -1\}$ then show that $6 \le \iint_A (2x^2 + 3y^2)\, dxdy \le 40$.

   (b) Evaluate $\iint_A yx(x+y)\, dxdy$ over region 'A' bounded by the curves $y = x^2$ and $y = -x$. 3+3=6

2. (a) Find area of region bounded by $y^2 = x + 2y$ and $y^2 = -x$, using double integration.

   (b) Find the volume of the solid bounded by the co-ordinate planes and the $2x + y + z = 2$; $2x + y + z = 4$.

   3+3=6

3. (a) Evaluate $\iiint\limits_V \frac{1-x^2-y^2-z^2}{1+x^2+y^2+z^2}\,dxdydz$, where

$$V = \left\{(x, y, z) \middle| x \geq 0, y \geq 0, z \geq 0, x^2 + y^2 + z^2 \leq 1\right\}.$$

(b) If $\vec{F} = (3x^2 + 6y)\hat{i} - 14yz\hat{j} + 20xz^2\hat{k}$ then evaluate $\int\limits_C \vec{F}\cdot d\vec{r}$, where C is the straight line joining (0, 0, 0) and (1, 1, 1). 3+3=6

4. (a) Verify Stokes' theorem for $\vec{F} = (2x - y)\hat{i} - yz^2\hat{j} - y^2z\hat{k}$ where S is the upper half surface of the sphere $x^2 + y^2 + z^2 = 1$ and "C" is its boundary.

(b) Evaluate $\iint\limits_S \vec{F}\cdot\hat{n}\, dS$ with the help of Gauss theorem for $\vec{f} = 6z\hat{i} + (2x + y)\hat{j} - x\hat{k}$ taken over the region "S" bounded by the surface of the cylinder $x^2 + z^2 = 9$ included between x = 0, y = 0, z = 0 and y = 8. 3+3=6

UNIT—II

5. (a) Use M_n-test to show that the sequence $\{f_n(x)\}$ converges uniformly on [0, 1/2] where $f(x) = \frac{x^n}{1 + x^n}\ \forall\ n \in N$.

(b) Examine for term by term integration of the series, the sum of whose 'n' term is $n^2x(1 - x)^n$, $0 \leq x \leq 1$. 3+3=6

6. (a) Show that the series $\sum_{n=1}^{\infty} \frac{1}{n^2 + n^4x^2}$ is uniformly convergent for all x and it can be differentiated term by term.

(b) Find the radius of convergence of power series

$\sum_{n=0}^{\infty} \frac{\lfloor 2n}{(\lfloor n)^2} x^n$. 4+2=6

7. (a) Show that $\log(1 + x) = x - \frac{x^2}{2} + \frac{x^3}{3} \ldots\ldots \forall x \in (-1, 1]$.

(b) Find the Fourier series expansion of $f(x) = x^2$ in $[-\pi, \pi]$. 3+3=6

8. (a) Find the Fourier expansion of $f(x) = x - x^3$ in $(-1, 1)$.

(b) Find the half range cosine series of $f(x) = x^2$ in $[0, \pi]$.

3+3=6