(i) Printed Pages: 4 Roll No. ....

(ii) Questions : 8

Sub. Code : 0 5 4 2

Exam. Code : 0 0 0 6

B.A./B.Sc. (General) 6^{th} Semester

(2054)

MATHEMATICS

Paper–II : Linear Algebra

Time Allowed : Three Hours] [Maximum Marks : 30

Note :— Attempt **FIVE** questions in all, selecting at least **TWO** questions from each unit. All questions carry equal marks.

UNIT—I

1. (a) Let V be the set of all $\sigma \times \sigma$ skew symmetric matrix over Real. If vector addition and Scalar multiplication be defined as usual addition of matrices and multiplication of Scalar with matrix, show that V is Vector space over Real. Also write dimension of V.

   (b) Show that intersection of two Vector Subspaces of Vector Space V is Vector Subspace of V, but Union may or may not be.

2. (a) Let $X = (1, 2, -1)$, $Y = (2, -3, 2)$, $Z = (4, 1, 0)$, $T = (-3, 1, -1)$ be four elements of Vector Space $V_3(\mathbb{R})$. If $L(s)$ denote Linear Span of Set S, then show that :

$L[\{X, Y\}] = L[\{Z, T\}]$.

(b) Define basis of Vector Space. If $\{v_1, v_2, v_3, \ldots v_{n-1}, v_n\}$ is basis of Vector Space V, show that

Set $B = \{v_1, v_1 + v_2, v_1 + v_2 + v_3, \ldots\ldots v_1 + v_2 + \rightarrow v_n\}$ is also Basis of Vector Space V.

3. (a) Find Basis and dimension of Subspace of $\mathbb{R}^4(\mathbb{R})$ spanned by Set $S = \{(1, 2, 3, 5), (2, 3, 5, 8), (3, 4, 7, 11), (1, 1, 2, 3)\}$.

(b) Let V and W be two Vector Spaces over same field $\mathbb{F}$. Define Null Space and Range Space of Linear Transformation $T : V \rightarrow W$. Show that Null Space of T is Vector Subspace of V.

4. (a) Find Linear Transformation $T : V_4(\mathbb{R}) \rightarrow V_3(\mathbb{R})$ whose Basis for Range Space of T is $\{(1, 2, -1), (3, -2, 5)\}$. Is T unique ?

(b) Let $T : \mathbb{R}^3 \rightarrow \mathbb{R}^3$ be linear transformation defined by $T(x, y, z) = (5x, 3x - y, 2x + 3y - z)$. Show that T is invertible and T^{-1}.

UNIT—II

5. (a) Let V and W be two finite dimensional Vector Spaces over field $\mathbb{F}$ and $T : V \to W$ be Linear Transformation. If B_1, B_2 be two ordered basis of V and W respectively, show that $[T; B_1, B_2]\,[v\,;\,B_1] = [T(v);\,B_2]$.

(b) If Matrix representation of Linear operator $T : \mathbb{R}^3 \to \mathbb{R}^3$ w.r.t. ordered basis B = {(1, 2, 2), (1, 1, 2), (1, 2, 1)}

is $\begin{bmatrix} 1 & 4 & -4 \\ -1 & -2 & 2 \\ 1 & -2 & 4 \end{bmatrix}$, find matrix representation of Linear operator T w.r.t. usual basis of $\mathbb{R}^3$.

6. (a) Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ be linear operator defined as $T(x, y, z) = (3x + z, -2x + y, -x + 2y + 4z)$. Let Matrix representation of T w.r.t. ordered basis

B_1 = {(1, 0, 1), (–1, 0, 1), (0, 1, 1)} and

B_2 = {(1, 0, 0), (0, 1, 0), (0, 0, 1)} be P and Q respectively. Show that P and Q are similar matrix.

(b) Find all eigen values and eigen vector of linear operator $T : \mathbb{R}^3 \to \mathbb{R}^3$ defined by

$T(x, y, z) = (2x + y, y - z, 2y + 4z)$.

Also find Algebraic multiplicity and Geometric multiplicity of each eigen value.

7\. (a) Is matrix $A = \begin{bmatrix} 3 & 10 & 5 \\ -2 & -3 & -4 \\ 3 & 5 & 7 \end{bmatrix}$ diagonalizable ? Justify.

(b) Show that all Eigen Values of unitary matrix are of unit modulus.

8\. (a) Verify Cayley-Hamilton theorem for Linear operator T on $\mathbb{R}^3$ defined by $T(x, y, z) = (2x - y, x + y + z, 2z)$.

(b) Define minimal polynomial of Matrix. Find Minimal polynomial of Matrix $A = \begin{bmatrix} 5 & -6 & -6 \\ -1 & 4 & 2 \\ 3 & -6 & -4 \end{bmatrix}$.