(i) Printed Pages: 4

(ii) Questions : 9

Roll No. ....

Sub. Code : | 3 | 7 | 0 | 9 |

Exam. Code : | 0 | 4 | 7 | 3 |

M.Sc. Physics 2nd Semester

(2054)

MATHEMATICAL PHYSICS—II

Paper : PHY–8021

Time Allowed : Three Hours] **[Maximum Marks : 60**

Note :—Attempt **FIVE** questions in all, selecting **ONE** question each from Units I to IV. Question No. **9** (Unit V) is compulsory.

UNIT—I

1. (a) Define conjugate class. Show that every element of a group belongs to one and only one class, and that the identity element forms a class by itself. Describe how the elements of the permutation group S_3 can be divided into classes. 6

   (b) Briefly discuss the three-dimensional rotation group. How many independent parameters does it have ? Obtain the general element of this group in terms of Euler angles. 6

2. (a) Define representation of a group. What are reducible and irreducible representations ? Deduce irreducible representations of C_{4v} group. 6

(b) List various applications of group theory. 6

UNIT—II

3. (a) $f(x) = x^4$, $-\pi < x < \pi$, show that :

$$\sum_{n=1}^{\infty} \frac{1}{n^4} = \frac{\pi^4}{90} = \zeta(4).$$ 4

(b) For the symmetrical finite step function

$$f(x) = \begin{cases} 1, & |x| < 1 \\ 0, & |x| > 1 \end{cases},$$

find Fourier cosine transform. Then taking the inverse cosine transform show that :

$$f(x) = \frac{2}{\pi} \int_0^{\infty} \frac{\sin \omega \cos \omega x}{\omega} d\omega,$$

and hence reobtain the original form of the symmetrical finite step function. 8

4. (a) The Fourier transform of the triangular pulse

$$f(x) = \begin{cases} h(1 - a|x|), & |x| < 1/a, \\ 0, & |x| > 1/a. \end{cases}$$ 6

(b) Using Laplace transforms solve the damped-oscillator equation

$$mX''(t) + bX'(t) + kX(t) = 0$$

for $b^2 < 4km$, given that $X(0) = X_0$, $X'(0) = 0$. 6

UNIT—III

5. (a) Transform the linear-oscillator equation $y'' + \omega^2 y = 0$ with the boundary conditions $y(0) = 0$, $y(b) = 0$ into an integral equation and identify the kernel. 6

(b) Using the Neumann series method solve the integral equation :

$$\phi(x) = x + \frac{1}{2}\int_{-1}^{1} (t - x)\phi(t)\, dt.$$ 6

6. (a) Show that $\varepsilon_{ijk}\varepsilon_{pqk} = \delta_{ip}\delta_{jq} - \delta_{iq}\delta_{jp}$. 3

(b) Explain inner and outer multiplications with examples. 2

(c) Show that the covariant derivative of a covariant vector is given by

$$V_{i;j} = \frac{\partial V_i}{\partial q^j} - V_k \Gamma^k_{ij}.$$ 7

UNIT—IV

7. (a) Discuss and compare the Euler and Runge-Kutta methods of numerical solution of differential equations. 8

(b) Briefly describe the Monte-Carlo technique. 4

8. (a) Briefly explain Central Limit Theorem. 5

(b) Briefly explain Poisson and Normal Distributions. 4

(c) If the probability that a man aged 70 will live to 80 is 0.6, what is the probability that 8 out of 10 men now aged 70 will live to 80 ? 3

UNIT—V

9. (a) Define isomorphism and homomorphism.

(b) Explain Gibbs phenomenon in the context of Fourier series.

(c) What are the advantages of Fourier representation over other representations ?

(d) Consider the integral

$$\phi(x) = f(x) + \lambda \int_a^b K(x, t)\rho(t)\phi(t)\, dt$$

with $K(x, t) = K(t, x)$. Show how this equation with non-symmetric kernel $K(x, t)\rho(t)$ can be transformed into one with symmetric kernel.

(e) Give the transformation property of a mixed tensor of rank two. What is contraction ?

(f) Briefly discuss the generations of random numbers.

6×2