(i) Printed Pages: 3 Roll No. ....

(ii) Questions : 8 **Sub. Code : 1 7 4 4 3**

Exam. Code : 0 0 0 5

B.A./B.Sc. (General) 5th Semester

(2124)

MATHEMATICS

Paper–I : Analysis–I

Time Allowed : Three Hours] **[Maximum Marks : 30**

Note :— Attempt **FIVE** questions in all by selecting **TWO** questions from each section. Each question carries 6 marks.

SECTION—A

1. (a) Prove that the interval [0, 1] is uncountable.

   (b) Prove that the set of rational numbers in [0, 1] is countable.

2. (a) Prove that $f : [a, b] \to \mathbb{R}$ is Riemann integrable if and only if for every $\epsilon > 0$, there exists some partition P such that $U(P; f) - L(P; f) < \epsilon$.

   (b) Let $f : [a, b] \to \mathbb{R}$ be Riemann integrable. Assume that there exists a function $F : [a, b] \to \mathbb{R}$ such that $F' = f$ on [a, b]. Prove that $\int_a^b f(t)dt = F(b) - F(a)$.

3. (a) If $f : [0, 1] \rightarrow \mathbb{R}$ is continuous, then f is Riemann integrable on [0, 1]. What can be said about the case of bounded functions on [0, 1] with only finitely many discontinuities ?

(b) If the Dirichlet function f, given as under, Riemann integrable on [0, 1] ?

$$f(x) = \begin{cases} 1 & \text{if } x \in Q \cap [0, 1] \\ 0 & \text{if } x \notin Q. \end{cases}$$

4. (a) State and prove the Mean Value Theorem for the Riemann integral.

(b) Establish the following well-known iterative formula for the gamma function $\Gamma(x + 1) = x\Gamma(x)$ for all $x > 0$.

SECTION—B

5. (a) State Dirichlet's test for improper integrals and discuss the convergence of

$$\int_1^{\infty} \frac{\arctan(9x)}{1+9x^3} dx.$$

(b) Does the integral $\int_1^{\infty} \frac{\sin^2 x}{x^2} dx$ converge ?

6. (a) State the general result about Frullani's integral and compute :

$$\int_1^{\infty} \frac{e^{-3x} - e^{-7'x}}{x} dx .$$

(b) If m $\in \mathbb{N}$, compute :

$$\int_0^1 \frac{(x^m + x^{-m})\log(1+x)}{x} dx .$$

7. (a) Evaluate the integral $\int_0^4 \frac{x}{x^2 - 4} dx$.

(b) Does absolute convergence imply conditional convergence for improper integrals ? Justify your answer.

8. (a) Prove that :

$$\int_0^{\pi/2} \sin x \log(\sin x)\, dx = \log\left(\frac{2}{e}\right).$$

(b) Does the integral $\int_0^{\infty} \cos\frac{1}{x} dx$ converge ?