(i) Printed Pages: 2 11/12 Roll No. ....

(ii) Questions : 8 **Sub. Code : 1 7 4 4 4**

Exam. Code : 0 0 0 5

B.A./B.Sc. (General) 5th Semester

(2124)

MATHEMATICS

Paper–II : Modern Algebra

Time Allowed : Three Hours] **[Maximum Marks : 30**

Note :—Attempt **FIVE** questions in all by selecting at least **TWO** from each section.

SECTION—A

1. (a) If G is a group, then the equations $ax = b$ and $ya = b$ have unique solution in G. 3

   (b) Show that if G is a group and $a^2 = e \ \forall \ a \in G$ then G is abelian. 3

2. (a) State and prove Lagrange's theorem on groups. 3

   (b) Prove that if H and K are subgroups of a group G then $H \cap K$ is also a subgroup of G. 3

3. (a) If G is a finite group of prime order then G is cyclic. 3

   (b) Show that A_4 has no subgroup of order six. 3

4. (a) Prove that every finite cyclic group of order n is isomorphic to Z/nZ. 3

(b) A group homomorphism $f : G \to G'$ is monomorphism iff $\ker f = \{e\}$. 3

SECTION—B

5. (a) Prove that a finite non-zero integral domain is a field. 3

(b) Prove that a ring R is commutative iff

$(a + b)^2 = a^2 + 2ab + b^2 \ \forall \ a, b \in R$ 3

6. (a) If I and J are ideals of a ring R then $I \cap J$ is also an ideal of R. 3

(b) Show that a division ring has no proper ideal. 3

7. (a) Find all units of $Z_7[x]$. 3

(b) Show that uZ is a maximal ideal of 2Z. 3

8. An ideal P of a commutative ring R is prime if and only if R/P is an integral domain. 6