(i) Printed Pages : 4 Roll No. ....

(ii) Questions : 8 **Sub. Code : 1 7 4 4 5**

Exam. Code : 0 0 0 5

B.A./B.Sc. (General) 5th Semester
(2124)
MATHEMATICS
Paper—III : Probability Theory

Time Allowed : Three Hours] **[Maximum Marks : 30**

Note :— Attempt **FIVE** questions in all, selecting at least **TWO** questions from each unit. All questions carry equal marks.

UNIT—I

1. (a) If $P(A) = \frac{1}{4}$, $P(B) = \frac{1}{3}$ and $P(A \cup B) = \frac{1}{2}$, evaluate $P(A|B)$, $P(B|A)$ and $P(A \cap B')$.

(b) A doctor is to visit a patient. From the past experience, it is known that the probabilities that he will come by bus, scooter, train any by other means of transport are $\frac{1}{5}, \frac{1}{10}, \frac{3}{10}$ and $\frac{2}{5}$ respectively. The probability that he will be late by bus, scooter and train are $\frac{1}{3}, \frac{1}{12}, \frac{1}{4}$ respectively but if he comes by other means of transport, he reaches in time.

When he arrives, he is late. Find the probability that :

(i) he comes by train

(ii) he comes by bus. 3,3

2. (a) The probability density function of a random variable X is given as :

$$f(x)=\begin{cases} x & 0<x<1 \\ 2-x & 1\le x<a \\ 0 & \text{otherwise} \end{cases}$$

Find :

(i) the value of a

(ii) the distribution function of X

(iii) $P(0.8 < X < 0.6a)$

(b) If expected value of random variable X exists, then expected value of X^2 also exists; comment. 3,3

3. (a) From the marks obtained by 120 students in Section A and B of a class, the following measures are obtained :

	$\overline{X}$	σ	Mode
Section A	46.83	14.8	51.57
Section B	47.83	14.8	47.07

Determine which distribution of marks is more skewed.

(b) Let the random variable X has probability density function given by $f(x) = \frac{1}{2}e^{-|x|}, -\infty < x < \infty$. Find the moment generating function and hence its mean and variance. 2,4

4. (a) If X as Poisson distribution with parameter λ, show $P(X = \text{even}) = \frac{1+e^{-2\lambda}}{2}$.

(b) A pair of dice is thrown 7 times. If getting a total of 7 is considered a success, what is the probability of :

(i) exactly 6 successes

(ii) at least 6 successes

(iii) at most 6 successes ? 3,3

UNIT—II

5. (a) If families are selected randomly in a certain thickly populated area and their monthly income in excess of 4000 is treated as exponential random variable with parameter $\lambda = \frac{1}{2000}$. What is the probability that 3 out of 4 families selected in the area have income in excess of Rs. 5,000 ?

(b) Let X be uniformly distributed over $(-\alpha, \alpha)$ where $\alpha > 0$. Find α so that :

(i) $P(X > 1) = \frac{1}{3}$

(ii) $P(X < \frac{1}{2}) = 0.8$

(iii) $P(|X| < 1) = P(|X| > 1)$ 3,3

6. Verify $f(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}, -\infty < x < \infty$ where $\sigma > 0$ is p.d.f. of normal variate and μ is its mean. 6

7. The Joint probability density function of two dimensional (X, Y) is given as :

$$f(x,y)=\begin{cases} x^2+\dfrac{xy}{3} & \text{for } 0<x<1, 0<y<2 \\ 0 & \text{otherwise} \end{cases}$$

Compute :

(i) $P\left(X>\frac{1}{2}\right)$

(ii) $P\left(Y<\frac{1}{2} \mid X>\frac{1}{2}\right)$ 6

8. (a) The coefficient of correlation is independent of change of scale and origin (X, Y) bivariate discrete random variable.

(b) Let $f(x,y)=\begin{cases} \frac{2}{3}(x+2y) & \text{for } 0<x<1,\ 0<y<1 \\ 0 & \text{otherwise} \end{cases}$

Find :

(i) $E(Y|X=x)$

(ii) $E(X|Y=y)$. 4,2