(i) **Printed Pages: 3** **Roll No. .....**

(ii) **Questions : 8** **Sub. Code :** | 1 | 7 | 2 | 4 | 1 |

Exam. Code : | 0 | 0 | 0 | 3 |

B.A./B.Sc. (General) 3rd Semester

(2124)

MATHEMATICS

Paper–I (Advanced Calculus–I)

Time Allowed : Three Hours] **[Maximum Marks : 30**

Note :—Attempt **FIVE** questions in all, selecting at least TWO questions from each section. All questions carry equal marks.

SECTION—A

1. (a) By using definition, prove that :

$$\lim_{(x, y) \to (1, 2)} (x^2 + y^2) = 5.$$ 3

(b) Show that the function defined by :

$$f(x, y) = \begin{cases} \dfrac{xy}{\sqrt{x^2 + y^2}} & , (x, y) \neq (0, 0) \\ 0 & , (x, y) = (0, 0) \end{cases}$$

is continuous at (0, 0). 3

2. Let $f(x, y) = xy\left(\dfrac{x^2 - y^2}{x^2 + y^2}\right)$, where $(x, y) \neq (0, 0)$ and $f(0, 0) = 0$. Show that $f_{xy}(0, 0) \neq f_{yx}(0, 0)$. 6

3. (a) If $x^y + y^x = (x + y)^{x+y}$, then by using partial derivatives, find $\frac{dy}{dx}$. 3

(b) If $z = x^3 - xy + y^3$ and $x = r \cos \theta$, $y = r \sin \theta$, find $\frac{\partial z}{\partial r}$ and $\frac{\partial z}{\partial \theta}$. 3

4. (a) If $\vec{r} = t^3\hat{i} + (2t^3 - \frac{1}{5t^2})\hat{j}$ then show that $\vec{r} \times \frac{d\vec{r}}{dt} = \hat{k}$. 3

(b) For $\phi = x^2y^3z^4$, find the directional derivative of ϕ at (2, 3, 1) in the direction making equal angles with x, y and z-axis. 3

SECTION—B

5. (a) If $u = \tan^{-1}\left(\frac{x^3 + y^3}{x - y}\right)$ then prove that :

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \sin 2u.$$ 3

(b) Use Taylor's theorem to expand $x^2y + 3y - 2$ in powers of $(x - 1)$ and $(y + 2)$. 3

6. (a) If $f(x, y) = \left(\sqrt{x^2 + y^2}, \tan^{-1}\frac{y}{x}\right)$, $x \neq 0$ then evaluate $J_f(1, 2)$. 3

(b) If $f(0) = 0$ and $f'(x) = \frac{1}{1 + x^2}$, prove without using method of integration, that $f(x) + f(y) = f\left(\frac{x + y}{1 - xy}\right)$. 3

7. (a) Find the envelope of the family of circles $(x - \alpha)^2 + y^2 = 4\alpha$; α being the parameter. 3

(b) Find the centre of curvature at any point (x, y) of the parabola $y^2 = 4ax$. Also find its evolute. 3

8. Find all the points of maxima and minima of the function $f(x, y) = x^3 + y^3 - 63(x + y) + 12xy$. Also discuss the saddle points (if any) of the function. 6