Exam.Code:0033
Sub. Code: 17959

2124

B.Sc. (Hons.) Biotechnology
First Semester
BIOT-103A-T: Mathematics

Time allowed: 3 Hours **Max. Marks: 67**

NOTE: *Attempt five questions in all, including Question No. I (Section-A) which is compulsory and selecting two questions each from Section B-C.*

x-x-x

Section A

1. (a) Evaluate $\int_0^1 (xe^x + \sin\frac{\pi x}{4})dx$. [3]

(b) $\lim_{x\to 0} \frac{\cos x}{(\pi - x)}$. [2]

(c) Find the general solution of the differential equation $x^5\frac{dy}{dx} = -y^5$. [2]

(d) Determine the degree and order of the differential equation $(\frac{ds}{dt})^4 + 3s\frac{d^2s}{dt^2} = 0$. [2]

(e) Find $\frac{dy}{dx}$ if $y = x^{-3}(5 - x)$ [2]

(f) Let $U = \{1, 2.3, 4, 5, 6, 7, 8, 9\}$, $A = \{1, 2, 3, 4\}$, $B = \{2, 4, 6, 8\}$, $C = \{3, 4, 5, 6\}$. Find $(A \cup B)'$ and $(A \cup C)'$. [2]

(g) Find the mean and median of the following data: 14, 15, 8, 13, 17, 11, 12, 20, 16. [2]

Section B

2. (a) Find the modulus and principal argument of the complex number $-1 + \sqrt{3}i$. [4]

(b) Find the value of $\frac{2+3i}{3-4i}$. [4]

(c) Express the complex number $(-\sqrt{3} + \sqrt{-2})(2\sqrt{3} - i)$ in the form $a + ib$. [5]

3. (a) Prove that $\sqrt{5}$ is an irrational number. [5]

(b) Evaluate $\lim_{x\to 2} \frac{3x^2 - x - 10}{x^2 - 4}$. [4]

(c) Find $\lim_{x\to 5} f(x)$, where $f(x) = |x| - 5$. [4]

4. (a) Differentiate the function $f(x) = (log\ x)^{\cos x}$ with respect to x. [4]

(b) Let $f, g : R \to R$ be defined by $f(x) = \sqrt{x}$ and $g(x) = sin(x - 2)$ then show that fog, gof are not equal. [4]

(c) Draw the graph of the function $f(x) = \frac{1}{x}, x \in \mathbb{R}$ and $x \neq 0$. [5]

P.T.O.

5. (a) Find the domain and range of the function $f(x) = \sqrt{x-1}$, where x is a real number. [4]

(b) Find all the points of discontinuity of f, where f is defined by

$$f(x) = \begin{cases} 2x+3 & \text{if } x \leq 2 \\ 2x-3 & \text{if } x > 2 \end{cases}$$ [5]

(c) Let $f : R \to R$ be defined by $f(x) = 4x + 3$. Show that f is invertible. Find the inverse of f. [4]

Section C

6. (a) Find $\frac{dy}{dx}$ if $y = e^{\sec^2 x} + 3\cos^{-1} x$. [5]

(b) Find the intervals in which the function $f(x) = 4x^3 - 6x^2 - 72x + 30$ is strictly (i) increasing and (ii) decreasing. [4]

(c) Find the local maximum and minimum, if any, of the given function $f(x) = x^3 - 6x^2 + 9x + 15$. [4]

7. (a) Evaluate $\int (x^2+1) log x dx$. [4]

(b) Find the area of the region bounded by the ellipse $\frac{x^2}{4} + \frac{y^2}{9} = 1$. [5]

(c) Evaluate $\int_0^{\pi} \frac{x}{1+\sin x} dx$. [4]

8. (a) Solve the following Linear Programming Problem graphically: Maximise $Z = 3x + 4y$ subject to the constrains: $x + y \leq 4, x \geq 0, y \geq 0$. [7]

(b) Draw a histogram and frequency polygon of the following frequency distribution: . [6]

Weight (in kg)	35-40	40-45	45-50	50-55	55-60	60-65
No. of workers	6	17	30	8	3	5

9. (a) Calculate mean, variance and standard deviation for the following distribution. [7]

Classes	30-40	40-50	50-60	60-70	70-80	80-90	90-100
Frequencies	3	7	12	15	8	3	2

(b) If the mean of the following frequency distribution is 28.25, find the value of p.

x_i	15	20	25	30	35	40
f_i	8	7	p	14	15	6

[6]