(i) Printed Pages: 4 Roll No. ....

(ii) Questions : 9 **Sub. Code : 2 6 1 0 1**

Exam. Code : 0 4 7 2

M.Sc. Physics 1st Semester

(2124)

MATHEMATICAL PHYSICS—I

Paper : PHY–8011

Time Allowed : Three Hours] **[Maximum Marks : 60**

Note :—(1) Attempt **FIVE** questions in all, taking **ONE** question each from Units I to IV.

(2) Unit–V is compulsory.

UNIT—I

1\. (a) Assuming that f(z) is analytic on and with in the closed contour C, obtain an expression for the n^{th} derivative $f^{(n)}(z_0)$ of f(z) at z_0. 4

(b) Using the calculus of residues, show that :

$$\int_{-\infty}^{\infty} \frac{\cos x}{x^2 + a^2}\, dx = \frac{\pi}{a} e^{-a}.$$ 5

(c) Find the analytic function f(z) = u(x, y) + iv(x, y) if $u(x, y) = x^3 - 3xy^2$. 3

2\. (a) If f(z) is a regular function of z, prove that :

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) |f(x)|^2 = 4\,|f'(x)|^2.$$ 4

(b) Show that the integral :

$$\int_{-\infty}^{\infty} \frac{\cos bx - \cos ax}{x^2} dx \; a > b > 0 = \pi(a-b).$$ 5

(c) Evaluate $\oint_C \frac{\sin^2 z \, dz}{(z-a)^4}$. 3

UNIT—II

3. (a) Find relation between beta and gamma function. 4

(b) Define Dirac delta function. Show that the sequence $\delta_n(x)$ based on the function $\delta_n(x) = 0, \; x < 0$ and

$$\delta_n(x) = \frac{ne^{-n^2x^2}}{\sqrt{\pi}}, x > 0$$ is a delta sequence. 4

(c) Show that for integer s, $\int_0^{\infty} x^{2s+1} e^{-ax^2} dx = \frac{s!}{2a^{s+1}}$. 4

4. (a) Obtain Legendre duplication formula :

$$\Gamma(2m) = 2^{2m-1}(\pi)^{\frac{-1}{2}} \Gamma(m) \Gamma\left(m + \frac{1}{2}\right).$$ 5

(b) Prove that :

$$\delta(x^2 - a^2) = \frac{1}{2a}[\delta(x-a) + \delta(x+a)], a > 0.$$ 4

(c) Evaluate $\int_{-1}^{+1} (1+x)^a (1-x)^b \, dx$ in terms of beta function.

3

UNIT—III

5. (a) Solve the Bessel differential equation :

$$x^2y''(x) + xy'(x) + (x^2 - n^2)\, y(x) = 0$$

using Frobenius method. Are the two solutions independent ? Discuss. 6

(b) Using the method of separation of variables, solve Laplace equation in spherical polar co-ordinates. 6

6. (a) Determine the steady state temperature distribution in thin plate boundary by the lines $x = 0$, $x = 1$, $y = 0$ and $y = \infty$; assuming that heat can't escape from either surface of the plate, the edges $x = 0$, $x = 1$, $y = \infty$, being kept at steady temperature $F(x)$. 6

(b) Find the eigen values and normalized eigen vectors of the following matrix :

$$\begin{bmatrix} 3 & 1 & 4 \\ 0 & 2 & 6 \\ 0 & 0 & 5 \end{bmatrix}.$$ 6

UNIT—IV

7. (a) Show that the orthonormal property of Hermite polynomial is :

$$\int_{-\infty}^{\infty} e^{-x^2} H_n(x) H_m(x)\, dx = 2^n n! \sqrt{\pi}\, \delta_{mn}.$$ 6

(b) Obtain the Rodrigue's formula for Legendre polynomials. Hence find $P_3(x)$. 6

8. (a) Deduce that $\int_0^\infty e^{-x} L_m(x) L_n(x)\, dx = \delta_{m,n}$. 6

(b) Show that when n is an integer :

$$J_n(x) = \frac{1}{\pi}\int_0^\pi \cos(n\theta - x\sin\theta)\, d\theta.$$ 6

UNIT—V

(Compulsory Question)

9. (a) Show that : $xJ'_n(x) = -nJ_n(x) + xJ_{n-1}(x)$.

(b) Explain dispersion relations of complex functions.

(c) Evaluate $\oint_C \frac{dz}{z(2z+1)}$ for the contour the unit circle.

(d) Prove that $\delta'(x) = \frac{-1}{x - x_0}\delta(x - x_0)$.

(e) Give the physical meaning of divergence of a vector.

(f) If A and B are idempotent matrices, then A + B will be idempotent if and only if AB = BA = 0.

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