(i) Printed Pages: 4 Roll No. ....

(ii) Questions : 9 Sub. Code : | 1 | 0 | 0 | 0 | 2 |

Exam. Code : | 5 | 0 | 0 | 1 |

Bachelor of Arts (FYUP) 1st Semester

(2124)

MATHEMATICS

Paper : Algebra & Trigonometry MATDSC1

(Common with B.Sc. 1st Sem. N.E.P.)

Time Allowed : Three Hours] **[Maximum Marks : 90**

Note :—Attempt **FIVE** questions in all including Q. No. **1** which is compulsory and selecting **ONE** question from each unit.

(Compulsory Question)

1. (a) Find the co-efficient of x^5 in the expansion of $(x + 3)^8$. 4

(b) If $a \cos^2\frac{C}{2} + c \cos^2\frac{A}{2} = \frac{3b}{2}$, then the sides of the triangle are in A.P. 4

(c) If $A = \begin{bmatrix} 3 & 4 \\ 1 & 2 \end{bmatrix}$, find the value of $3|A|$. 2

(d) If the matrix $A = \begin{bmatrix} 0 & a & -3 \\ 2 & 0 & -1 \\ b & 1 & 0 \end{bmatrix}$ is skew-symmetric, find the values of a and b. 4

(e) Determine the rank of following matrix $\begin{bmatrix} 2 & 3 & 3 \\ 3 & 6 & 12 \\ 2 & 4 & 8 \end{bmatrix}$. 4

UNIT—I

2. (a) Use Principle of Mathematical induction to prove that

$$\frac{1}{1.2.3} + \frac{1}{2.3.4} + \frac{1}{3.4.5} + \ldots + \frac{1}{n(n+1)(n+2)} = \frac{n(n+3)}{4(n+1)(n+2)}.$$

(b) By mathematical induction, prove that

$$1^2 + 3^2 + 5^2 + \ldots + (2n-1)^2 = \frac{n(2n-1)(2n+1)}{3} \ \forall n \in N.$$

9,9

3. (a) The co-efficients of $(r-1)^{th}$, r^{th} and $(r+1)^{th}$ terms in the expansion of $(x+1)^n$ are in the ratio 1 : 3 : 5. Find both n and r.

(b) Assuming x to be so small that x^2 and higher powers of x can be neglected, show that

$$\frac{\left[1 + \frac{3x}{4}\right]^{-4} [16-3x]^{\frac{1}{2}}}{(8+x)^{\frac{2}{3}}} = 1 - \frac{305}{96}x.$$

9,9

UNIT—II

4. (a) Prove that $(\sqrt{3}+i)^n + (\sqrt{3}-i)^n = 2^{n+1}\cos\frac{n\Pi}{6}$ where n is a +ve integer.

(b) Find all the values of $\left(\frac{1}{2}+i\frac{\sqrt{3}}{2}\right)^{\frac{3}{4}}$ and show that the continued product of all the values is 1. 9,9

5. (a) If ξ is a primitive 8^{th} root of unity, find $(x-\xi)(x-\xi^3)(x-\xi^5)(x-\xi^7)$.

(b) Prove that :

$$\cos^6\theta \sin^4\theta = 2^{-9}[\cos 10\theta + 2\cos 8\theta - 3\cos 6\theta - 8\cos 4\theta + 2\cos 2\theta + 6].$$ 9,9

UNIT—III

6. (a) Prove that $\begin{vmatrix} b+c & a & a \\ b & c+a & b \\ c & c & a+b \end{vmatrix} = 4abc.$

(b) Prove that :

$$\begin{vmatrix} 3a & -a+b & -a+c \\ -b+a & 3b & -b+c \\ -c+a & -c+b & 3c \end{vmatrix} = 3(a+b+c)(ab+bc+ca).$$

9,9

7. (a) Express the matrix $A = \begin{bmatrix} 2-i & 3 & 1+i \\ -5 & 0 & -6i \\ 7 & i & -3+2i \end{bmatrix}$ as the sum of a Hermitian and a Skew-Hermitian matrix.

(b) Find the value of x so that matrix $\begin{bmatrix} x+p & q & r \\ p & x+q & r \\ p & q & x+r \end{bmatrix}$ is of rank 3. 9,9

UNIT—IV

8. (a) Check for the linear dependence of the following system of vectors :

$u = (1, -1, 1)$, $v = (2, 1, 1)$, $w = (3, 0, 2)$.

If dependent, find the relation between them.

(b) Find the values of k so that the equations $x - 2y + z = 0$, $3x - y + 2z = 0$, $y + kz = 0$ have :

(i) Unique solution

(ii) Infinitely many solutions. Also find solutions for these values of k. 9,9

9. (a) Find the characteristic roots and their corresponding characteristic vectors for the matrix $\begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ -7 & 5 & 1 \end{bmatrix}$.

(b) State and prove Cayley-Hamilton theorem. 9,9