(i) Printed Pages: 4 Roll No. ....

(ii) Questions : 9

Sub. Code : 3 7 0 2

Exam. Code : 0 4 7 2

M.Sc. Physics 1st Semester

(2122)

MATHEMATICAL PHYSICS—I

Paper : PHY-8011

Time Allowed : Three Hours] **[Maximum Marks : 60**

Note :—Attempt **five** questions in all, including Question No. 9 (Unit V) which is compulsory and selecting **one** question each from Units I–IV.

UNIT–I

1. (a) State the Cauchy's integral theorem and discuss the Goursat's proof of the theorem.

(b) Show that for any function of complex variable $f(z) = u + iv$, we can write :

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)|f(z)|^2 = 4|f'(z)|^2$$

(c) Evaluate residue of the function :

$$\frac{ze^{iz}}{z^2 - a^2}; a > 0$$

4,4,4

2. (a) Using Cauchy residue theorem, evaluate the integral :

$$\int_{-\infty}^{\infty} \frac{dx}{(1 + x^2)^3} dx$$

(b) Solve the following integration for multiple valued function :

$$\int_0^{\infty} \frac{x^{\alpha-1}}{1+x} dx = \frac{\pi}{\sin \alpha\pi} \quad \text{if } 0 < \alpha < 1$$ 6,6

UNIT–II

3. (a) Show that derivative of Dirac-delta function satisfy the relations :

$$\delta'(x - x_0) = -\frac{\delta(x - x_0)}{(x - x_0)},$$

where δ is the Dirac's delta function.

(b) Prove the following duplication formula for gamma function :

$$\Gamma(2m) = \frac{s^{2m-1}}{\sqrt{\pi}} \Gamma(m) \Gamma\left(m + \frac{1}{2}\right)$$ 6,6

4. (a) Using the property of gamma function, prove that :

$$\int_0^{\infty} x^{2s+1} e^{-ax^2} dx = \frac{s!}{2a^{s+1}}$$

(b) Prove that :

$$\int_0^{\pi/2} \sin^p \theta \sin^q \theta = \frac{\Gamma\left(\frac{p+1}{2}\right)\Gamma\left(\frac{q+1}{2}\right)}{2\Gamma\left(\frac{p+q+2}{2}\right)}$$

(c) Obtain the relation between beta and gamma function.

4,4,4

UNIT–III

5. (a) Obtain the series solution of $\frac{d^2y}{dx^2} + y = 0$, using power series of Frobenius method.

(b) Find the eigen values and normalized eigen vectors for the matrix :

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

6,6

6. (a) Solve the Helmholtz differential equation : $\nabla^2\Psi + k^2\Psi = 0$, using the separation of variable technique.

(b) Investigate the singularities associated with Bessel's differential equation :

$$x^2y'' + xy' + (x^2 - n^2)y = 0$$

(c) Express the perturbed electrostatic potential of a conducting sphere placed in a uniform electric field in term of Legendre's polynomial. 4,4,4

UNIT–IV

7. (a) Show that $\int_0^\infty e^{-ax} J_0(bx)\,dx = \frac{1}{\sqrt{a^2 + b^2}}$; $a, b \geq 0$.

(b) Prove the following recurrence relations :

(i) $P'_{n+1}(x) - P'_{n-1}(x) = (2n+1)\,P_n(x)$

(ii) $P'_{n+1}(x) - xP'(x) = nP_n(x)$

where $P_n(x)$ is the Legendre's polynomial of n^{th} order.

6,6

8. (a) Obtain the Orthogonality conditions for associated Legandre's polynomials.

(b) Derive the expression of Rodrique's formula

$$H_x(x) = (-1)^n e^{x^2} \frac{d^n}{dx^n}(e^{-x^2})$$ for Hermite's polynomial.

6,6

UNIT—V

9. (a) Define branch points and branch cuts in any complex plane.

(b) Write down the Cauchy-Riemann's (CR) condition for analytic function in Polar form.

(c) Prove that Dirac Delta function $\delta(x)$ is symmetric in nature.

(d) Show that the value of $P_n(1) = 1$.

(e) Prove that symmetry of solution is direct consequences of the order of differential equations.

(f) Show that $\int_0^\infty e^{-x^4} dx = \Gamma\left(\frac{5}{4}\right)$. 2,2,2,2,2,2