(i) **Printed Pages: 3** **Roll No. .....**

(ii) **Questions : 8** **Sub. Code : 0147**

Exam. Code : 0002

B.A./B.Sc. (General) 2nd Semester

(2042)

MATHEMATICS

Paper : III (Theory of Equations)

Time Allowed : Three Hours] **[Maximum Marks : 30**

Note :— (1) Attempt **FIVE** questions in all by selecting at least **TWO** questions from each unit.

(2) All questions carry equal marks.

UNIT—I

1. (a) Show that $x^4 + 4x^3 - 2x^2 - 12x + 9 = 0$ has a multiple root and hence solve it completely.

(b) If $\alpha_1, \alpha_2, \ldots\ldots \alpha_n$ are n roots of a nth degree polynomial equation $f(x) = 0$, then prove

$$\frac{1}{\alpha_1 - \alpha} + \frac{1}{\alpha_2 - \alpha} + \ldots\ldots + \frac{1}{\alpha_n - \alpha} = \frac{-f'(\alpha)}{f(\alpha)}, \quad f(\alpha) \neq 0.$$

3,3

2. (a) Solve $x^4 + 4x^3 + 6x^2 + 4x + 5 = 0$ if i is its root.

(b) Solve $x^4 - 6x^3 + 11x^2 - 18x + 9 = 0$, if ratio of two roots of it is equal to ratio of other two roots. 3,3

3. (a) Solve $3x^3 + 10x^2 + 40x + 192 = 0$, given that all the three roots are of equal moduli.

(b) If reciprocal of every root of $x^3 + x^2 + ax + b = 0$ is also a root, then prove $a = b = 1$ or $a = b = -1$. Also find roots of it. 3,3

4. (a) Diminish roots of equation $a_0x^3 + 3a_1x^2 + 3a_2x + a_3 = 0$ by h and find the condition that 2nd and 3rd terms are removed simultaneously. Hence solve the equation $x^3 + 6x^2 + 12x - 19 = 0$.

(b) If α, β, γ are roots of $x^3 + px + q = 0$. Form an equation having roots $\frac{\beta^2+\gamma^2}{\alpha^2}, \frac{\gamma^2+\alpha^2}{\beta^2}, \frac{\alpha^2+\beta^2}{\gamma^2}$. 3,3

UNIT—II

5. (a) Use Newton's method of divisors to find the integral roots of equation $3x^4 - 23x^3 + 35x^2 + 31x - 30 = 0$.

(b) If $\frac{p}{q}$ is a rational root of equation :

$a_0x^n + a_1x^{n-1} + a_2xn^{-2} + \ldots + a_n = 0$, where

$a_0, a_1, \ldots a_n \in Z$, $a_n \neq 0$, $p, q \in Z$, $(p, q) = 1$ then prove p/a_n, q/a_0. 3,3

6. (a) Solve $x^3 - 3x^2 + 12x + 16 = 0$ by Cardon's method.

(b) Apply Trigonometric method to prove the equation $x_3 - 12x + 8 = 0$ has roots $4\cos\frac{2\pi}{9}$, $4\cos\frac{4\pi}{9}$, $4\cos\frac{8\pi}{9}$. 3,3

7. (a) Discuss the nature of roots of the equation

$$x^3 + 6x^2 + 9x + 4 = 0.$$

(b) Solve $2x^4 + 6x^3 - 3x^2 + 2 = 0$ by expressing it as product of two quadratic factors. 3,3

8. (a) Explain Ferrari's method to solve a biquadratic equation $a_0x^4 + 4a_1x^3 + 6a_2x^2 + 4a_3x + a_4 = 0$.

(b) Solve by Ferrari's method the equation

$$x^4 - 10x^3 + 35x^2 - 50x + 24 = 0.$$ 3,3