(i) **Printed Pages: 4** Roll No. ....

(ii) **Questions : 8** **Sub. Code :** | 0 | 5 | 4 | 2 |

Exam. Code : | 0 | 0 | 0 | 6 |

B.A./B.Sc. (General) 6th Semester

(2042)

MATHEMATICS

Paper : II (Linear Algebra)

Time Allowed : Three Hours] **[Maximum Marks : 30**

Note :— Do any **FIVE** questions, selecting at least **TWO** questions from each unit. Each question is of **6** marks.

UNIT—I

1. (a) State and prove the necessary and sufficient condition for a non-empty subset of Vector Space is Vector Subspace.

(b) Define Linear Span of Subset of Vector Space. Let $x = (1, 2, 1)$, $y = (3, 1, 5)$, $z = (3, -4, 7)$ be three elements in $\mathbb{R}^3$, show that

$L[\{x, y\}] = L[\{x, y, z\}]$. 3+3

2. (a) Let W_1, W_2 be two finite dimensional Vector Subspaces of Vector Space $V(\mathbb{F})$, show that

$\dim(W_1 + W_2) = \dim W_1 + \dim W_2 - \dim(W_1 \cap W_2)$.

(b) Find Basis and dimension of Vector Subspace W of $\mathbb{R}^4$ generated by set $S = \{(1, 2, 3, 5), (2, 3, 5, 8), (3, 4, 7, 1), (1, 1, 2, 3)\}$. Hence extend the basis to from basis of $\mathbb{R}^4$. 3+3

3. (a) State Rank-Nullity theorem and verify it for Linear operator $T - \mathbb{R}^3 \rightarrow \mathbb{R}^3$ given by

$$T(x, y, z) = (x + 2y - z, y + z, x + y - 2z).$$

(b) Let $V = \{A; A = [a_{ij}]_{n \times n}, a_{ij} \in \mathbb{R}\}$ be Vector Space over Real. Show that W, the set consisting of all Skew Symmetric matrix is Vector Subspace of V. Also find dimension of W. 3+3

4. (a) Define Non-Singular Transformation. Prove that Linear Transformation $T : V(\mathbb{F}) \rightarrow W(\mathbb{F})$ is non-singular if and only if the set of images of Linear Independent Set is Linear Independent.

(b) Let $T : V_3(\mathbb{R}) \rightarrow V_3(\mathbb{R})$ be linear operator defined by

$$T(x, y, z) = (3x , x - y, 2x + y + z).$$

If $f(t) = t^3 - 3t^2 - t + 3$ then $f(T)(x, y, z)$. 3+3

UNIT—II

5. (a) Let $T : \mathbb{R}^2 \to \mathbb{R}^3$ be Linear Transformation defined as $T(x, y) = (3x - 2y, 2x + y, x + 4y)$. Let $B_1 = \{(1, 1), (0, 2)\}$ and $B_2 = \{(1, 1, 0), (1, 0, 1), (0, 0, 1)\}$ be ordered basis of $\mathbb{R}^2$ and $\mathbb{R}^3$ and respectively, show that

$$[T; B_1, B_2]\,[v; B_1] = [T(v); B_2]\ \forall\ v \in \mathbb{R}^2.$$

(b) If the matrix of Linear operator T on $\mathbb{R}^3$ relative to usual basis of $\mathbb{R}^3$ is $\begin{bmatrix} 1 & 1 & -1 \\ -1 & 1 & 1 \\ 1 & -1 & 1 \end{bmatrix}$ then find matrix of Linear operator T relative to basis.

$\{(1, 2, 2), (1, 1, 2), (1, 2, 1)\}$ of $\mathbb{R}^3$. 3+3

6. (a) Show that Eigen value of unitary matrix are of absolute value 1.

(b) Find all Eigen value and Basis of each eigen space of linear operator $T : \mathbb{R}^3 \to \mathbb{R}^3$ defined by

$$T(x, y, z) = (2x + y, y - z, 2y + 4z).$$ 3+3

7. (a) Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ be Linear operator given by

$$T(x, y, z) = (5x - 6y - 6z, -x + 4y + 2z, 3x - 6y - 4z).$$

Find minimal polynomial of T.

(b) Show that Eigen Vectors corresponding to distinct Eigen Values of Linear Operator are Linear Independent.

3+3

8. (a) Using Cayley-Hamilton theorem find T^{-1} where $T : \mathbb{R}^3 \rightarrow \mathbb{R}^3$ is given by $T(x, y, z) = (3x - z, 2x + y, -x + 2y + 4z)$.

(b) Find the Linear Transformation $T : \mathbb{R}^2 \rightarrow \mathbb{R}^3$ determined by matrix $\begin{bmatrix} -2 & -6 \\ 3 & 2 \\ 2 & 6 \end{bmatrix}$ with respect to ordered basis $B_1 = \{(1, 2), (0, 3)\}$ and $B_2 = \{(1, 1, 0), (0, 1, 1), (1, 1, 1)\}$ of $\mathbb{R}^2$ and $\mathbb{R}^3$ respectively. 3+3