Exam Code: 0001
Sub. Code: 0044

2031
B.A./B.Sc. (General) First Semester
Mathematics
Paper – II: Calculus - I

Time allowed: 3 Hours

Max. Marks: 30

NOTE: *Attempt five questions in all, selecting atleast two questions from each Unit.*

x-x-x

UNIT – I

I. a) Prove that between any two distinct real numbers, there exist infinitely many real numbers.

b) Solve the inequality $\frac{x-2}{x+2} > \frac{2x-3}{4x-1}$ (2x3)

II. a) If |x - 5| < 3, then prove that $\frac{1}{4} < \frac{x^2+2x-5}{x+4} < \frac{13}{2}$.

b) Show that $\underset{x \to o}{Lt} \sin\frac{1}{x}$ does not exist. (2x3)

III. a) If F is a continuous function on [a,b], then prove that f attains its least upper bound and greatest lower bound in [a,b]

b) Determine constants a and b so that the function of defined below is continuous

$$f(x) = \begin{cases} 13, & if \quad x \le 2 \\ ax^2+bx+1, & if \quad 2 < x < 3 \\ 17-ax, & if \quad x \ge 3 \end{cases}$$ (2x3)

IV. a) Find values of a, b, c if $\underset{x \to 0}{lt} \frac{ae^x - b\cos x + ce^{-x}}{x\sin x} = 2$

b) Evaluate $\underset{x \to 0}{lt} \left[\frac{2(\cosh x - 1)}{x^2}\right]^{\frac{1}{x^2}}$ (2x3)

P.T.O.

UNIT – II

V. a) State and prove Lagrange's mean value theorem.

b) Use Cauchy's mean value theorem to prove that $\underset{n \to \infty}{Lt}\ n\left(x^{\frac{1}{n}} - 1\right) = \log x, x > 1.$

(2x3)

VI. a) Use Maclaurin's theorem to show that

$$\log(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \text{.............} + \frac{(-1)^{n-1}}{n}\frac{x^n}{(1+\theta x)^n}, 0 < \theta < 1$$

b) If $y = \frac{x\sqrt{x^2+a^2}}{2} + \frac{a^2}{2}\sinh^{-1}\left(\frac{x}{a}\right)$, then prove that $\left(\frac{dy}{dx}\right)^2 = x^2 + a^2$ (2x3)

VII. a) Prove that the functions $2\tanh^{-1}\left(\tan\frac{x}{2}\right)$ and $\cosh^{-1}$ (sec x) can differ only by a constant.

b) Use Taylor's theorem to express the polynomial $2x^3 + 7x^2 + x - 6$ in powers of $x - 2$. (2x3)

VIII. a) Let $f(x) = \begin{Bmatrix} x^2\sin\frac{1}{x}, & x \neq 0 \\ 0, & x = 0 \end{Bmatrix}$. Show that f' (x) is not continuous at x = 0 and f'' (x) does not exist at x = 0.

b) Prove that $\frac{d^n}{dx^n}\left(\frac{\log x}{x}\right) = \frac{(-1)^n \underline{|n}}{x^n + 1}[\log x - 1 - \frac{1}{2} - \frac{1}{3} \text{........} - \frac{1}{n}]$ (2x3)

x-x-x