Exam Code: 0001
Sub. Code: 0045

2041

B.A./B.Sc. (General) First Semester

Mathematics

Paper – III: Trigonometry and Matrices

Time allowed: 3 Hours

Max. Marks: 30

NOTE: *Attempt five questions in all, selecting atleast two questions each Unit.*

x-x-x

UNIT – I

I. a) If α, β be roots of $t^2 - 2t + 2 = 0$, then prove

$$\frac{(x+\alpha)^n + (x+\beta)^n}{\alpha+\beta} = \frac{\cos n\phi}{\sin^n \phi}, \quad x+1 = \cot\phi$$

b) Find nth roots of unity and prove that sum of their pth power always vanishes, unless p is multiple of n & in that case sum is n, where p ∈ z. (2x3)

II. a) Solve $x^6 - x^5 + x^4 - x^3 + x^2 - x + 1 = 0$ using De Moiver's theorem.

b) Express $\cos^5\theta \sin^7\theta$ in a series of sines of multiples of θ. (2x3)

III. a) For any integer k, evaluate $\sum_{r=1}^{n}\left[ExP\left(\frac{2\pi ir}{n}\right)\right]^k$

b) Separate into real and imaginary parts log log (x + iy) (2x3)

IV. a) If x + iy = tan (A + iB), prove that $x^2 + y^2 - 2y \coth 2B + 1 = 0$.

b) Find sum of n terms the series cos α + c cos (α+β) + c^2 cos (α + 2β) + …….. n terms. Also deduce the sum of infinity if k|<|. (2x3)

UNIT – II

V. a) If A is a non-sigular matrix, the prove f(a) = f(adj A) = f (A^{-1})

b) Prove every square matrix over C can be uniquely expressed as P + iQ, P, Q are Hermitian matrices. (2x3)

P.T.O.

VI. a) For the matrix $A = \begin{bmatrix} 1 & 2 & 0 & -1 \\ 3 & 4 & 1 & 2 \\ -2 & 3 & 2 & 5 \end{bmatrix}$. Find non-singular matrices, P, Q, s.t. PAQ is in normal form. Also rank of A.

b) For what value of λ, equation $-x + 2y + z = 0$, $3x - y + 2z = 0$, $y + \lambda z = 0$ have non-trivial solution. (2x3)

VII. a) Show that the equations $x + 2y - z = 3$, $3x - y + 2z = 1$, $2x - 2y + 3z = 2$, $x - y + z = -1$ are consistent. Also find solution.

b) If λ is an eigen value of a non-singular matrix A. then prove $\frac{|A|}{\lambda}$ is an eigen value of adj. A. (2x3)

VIII. a) If $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 3 & 1 & 2 \end{bmatrix}$, verify Cayley-Hamilton theorem and hence find adj A.

b) Find an invertible matrix P s.t. $P^{-1}AP$ is a diagonal matrix

$$A = \begin{bmatrix} 1 & 1 & 2 \\ -1 & 2 & 1 \\ 0 & 1 & 3 \end{bmatrix}$$

(2x3)

x-x-x