Exam.Code:0005
Sub. Code: 0444

2021

B.A./B.Sc. (General), Fifth Semester

Mathematics

Paper – II: Modern Algebra

Time allowed: 3 Hours | Max. Marks: 30

***NOTE**: Attempt five questions in all, selecting atleast two questions from each Unit.*

x-x-x

UNIT – I

I. a) If a and b be any two elements of group G such that $ab == ba$ and (0(a), (0(b)) = 1, then show that O(ab) = O(a)O(b).

b) Find out order of elements of Q_8, quarternions group. Is Q_8 abelian? Justify. (3,3)

II. a) State and prove Lagrange's theorem. Is converse true?

b) Let H be a subgroup of G Show that $O(H) = O(x^{-1}Hx), \forall x \in G$. (3, 3)

III. a) Let K be a normal subgroup of a group G and L be any subgroup of G, then prove that

$$LK/K \cong L/L \cap K$$

b) Find all subgroups of $Z/21Z$ (4,2)

IV. a) Prove that the subset A_n consisting of all the even permutations in S_n is a normal subgroup of index 2.

b) Let p = (1 2 3) and q = (1 3 2) be two permutation on the set {1,2, 3}. Calculate pqp^{-1}. (4, 2)

UNIT – II

V. a) Prove that a commutative ring with unity is a field iff it does not have any proper ideal.

b) Show that < 4 >, ideal generated by 4 is maximal ideal in ring 2Z of even integers. (4,2)

P.T.O.

VI. a) Show that a ring R is commutative iff $(a + b)^2 = a^2 + b^2 + 2ab$ for all $a, b \in R$.

b) Find the field of quotients of the integral domain $Z[\sqrt{2}]$. (3, 3)

VII. a) Prove that every field is integral domain. Is converse true? Justify.

b) Let $f: R \rightarrow R'$ be an onto homomorphism function. Then prove that

$$R/Kerf \cong R'$$ (3,3)

VIII. a) If R is an integral domain then prove that R[x], the ring of polynomials over R is also an integral domain.

b) Let R be an integral domain and f(x), $g(x) \in R[x]$. Prove that

deg (f(x).g(x))=deg(f(x))+deg(g(x)). (3,3)

x-x-x