

2031

M.Sc. (Bio-Informatics) First Semester
MBIN-8002: Mathematics

Time allowed: 3 Hours

Max. Marks: 60

NOTE: Attempt five questions in all, including Question No. I which is compulsory and selecting atleast one question from each Section.

x-x-x

I (a) Find the integral solution of $|2 + i|^x = 5^x$.

I (b) Find the sum of multiple of 3 upto 100.

I (c) Find the coefficient of x^3 in the expansion of $(1+x)^{-3}$.

I (d) $\lim_{x \rightarrow \frac{\pi}{2}} \frac{\cos x}{\frac{\pi}{2} - x}$

I (e) Differentiate $x^{\sqrt{x}}$ w.r.t. x .

I (f) Evaluate $\int_0^8 (x-5)dx$.

I (g) Find $\det(A)$, if

$$A = \begin{bmatrix} 1 & 2 & 0 \\ -2 & 4 & 3 \\ 0 & 8 & 5 \end{bmatrix}$$

I. (h) Find the value of λ such that the vectors $\vec{a}$ and $\vec{b}$ are perpendicular, where $\vec{a} = \lambda\hat{i} + 2\hat{j} + \hat{k}$, $\vec{b} = 5\hat{i} - 9\hat{j} + 2\hat{k}$.

(8 × 1.5)

Section A

II (a) If α and β are different complex numbers with $|\beta|^2 = 1$ then find $|\frac{\beta-\alpha}{1-\bar{\alpha}\beta}|$. (4)

II (b) Express $(-2 - \frac{1}{3})^3$ of the complex number in the form of $a + ib$. (4)

II (c) In a group of 65 people, each like at least one of cricket and tennis, 40 likes cricket, 10 likes both cricket and tennis. How many likes tennis only and not cricket? How many likes tennis? (4)

III (a) If the sum of first p terms of an A.P. is equal to the sum of the first q terms then find the sum of the first $p+q$ terms. (4)

P.T.O.

(2)

III (b) The sum of first three terms of an G.P. is 16 and the sum of the next three terms 128. Determine the first term and the common ratio. (4)

III (c) Find the tenth term in the expansion of $(1+x)^{\frac{7}{2}}$. (4)

Section B

IV (a) Evaluate $\lim_{x \rightarrow 4} \frac{3 - \sqrt{5+x}}{1 - \sqrt{5-x}}$. (4)

IV (b) If $x'' + y'' = 2$, find $\frac{dy}{dx}$. (4)

IV (c) The Base radius of a cylindrical vessel full of oil is 30 cm. Oil is drawn at the rate of $27000 \text{ cm}^3/\text{min}$. find the rate at which the level of oil is falling in the vessel. (4)

V (a) Evaluate $\int \frac{\sin x}{1 + \sin x} dx$. (4)

V (b) Prove that $\int_0^1 \sin^{-1} \frac{2x}{1+x^2} dx = \frac{\pi}{2} - \log 2$. (4)

V (c) Solve the differential equation, $\frac{dy}{dx} + 2y = 4x$. (4)

Section c

VI (a) Prove that the $\det(A)$ is independent of θ , if

$$A = \begin{bmatrix} x & \sin \theta & \cos \theta \\ -\sin \theta & x & 1 \\ \cos \theta & 1 & x \end{bmatrix} \quad (4)$$

VI (b) For what value of r , does the equations $x+y+4z = 6$, $x+2y-2z = 6$, $rx+y+z = 6$ have unique solution. (4)

VI (c) Find A^{-1} , if

$$A = \begin{bmatrix} 4 & 2 & 3 \\ 8 & 5 & 2 \\ 12 & -4 & 5 \end{bmatrix} \quad (4)$$

V II (a) Construct truth table for the following statements: $[p \wedge (q)] \rightarrow r$. (4)

VII (b) Determine area of the parallelogram whose adjacent sides are $\hat{i} - \hat{j} + 3\hat{k}$ and $2\hat{i} - 7\hat{j} + \hat{k}$. (4)

VII (c) Show that the vectors $2\hat{i} - \hat{j} + \hat{k}$, $\hat{i} - 3\hat{j} - 5\hat{k}$ and $3\hat{i} - 4\hat{j} - 4\hat{k}$ form the sides of a right angle triangle. (4)