Exam.Code:0472
Sub. Code: 3702

2031
M.Sc. (Physics)
First Semester
PHY-8011: Mathematical Physics – I

Time allowed: 3 Hours

Max. Marks: 60

***NOTE**: Attempt five questions in all, including Question No. 9 (Unit-V) which is compulsory and selecting one question each from Unit I - IV.*

x-x-x

UNIT-I

1. (a) Evaluate residue of the function $f(z) = \frac{e^z}{z^2+a^2}$ at its singularity.
   (b) Construct the function of complex variable $f(z) = u + iv$, if the real part $u = -r^3 sin3\theta$.
   (c) Evaluate $\int_c \frac{dz}{z^2-1}$, where c is a circle $x^2 + y^2 = 4$. (4, 4, 4)
2. (a) State and prove the Cauchy's integral theorem.
   (b) Apply calculus of residues to show that

$$\int_{-\infty}^{\infty} \frac{\cos x}{x^2+a^2} dx = \frac{\pi}{a} \pi^{-a} , a > o$$ (6, 6)

UNIT-II

3. (a) Prove that, $\delta(x^2 - a^2) = \frac{1}{2a}[\delta(x-a) + \delta(x+a)]$ for $a > 0$.
   (b) Using the property of gamma function, prove that

$$\int_0^{\pi/2} \sqrt{(\tan x)}\, dx = \frac{\Gamma\left(\frac{1}{4}\right)\Gamma\left(\frac{3}{4}\right)}{2}$$

   (c) Show that $\int_0^{\infty} e^{-x^2} dx = \frac{\sqrt{\pi}}{2}$ (4, 4, 4)
4. (a) Give various definitions of Γ function and show that Weistrass's form leads to an important identity

$$\Gamma(z).\Gamma(1-z) = \frac{\pi}{sin\, \pi z}$$

   (b) Prove that,

$$\delta'(x - x_o) = -\frac{\delta(x-x_o)}{(x-x_o)}$$, where δ is the Dirac's delta function. (6, 6)

UNIT-III

5. (a) Find the eigenvalues and normalized eigen vectors of the following matrix:

$$\begin{bmatrix} 3 & 1 & 4 \\ 0 & 2 & 6 \\ 0 & 0 & 5 \end{bmatrix}$$

   (b) Solve the Legendre's differential equation $(1-x^2)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + n(n+1)y = 0$, using power series method. (6, 6)

P.T.O.

6. (a) Solve two-dimensional heat flow equation with certain initial boundary conditions using separation of variable technique.
   (b) Give details about power series or Frobenius method to solve the linear differential equation about ordinary and regular singular point.

(6, 6)

UNIT-IV

7. (a) Obtain the orthogonality condition for the Bessel function.
   (b) Derive Rodrigue's formula for Legendre's polynomial. Using it, find the values of $P_0(x)$ and $P_1(x)$, and show their variation with x.

(6, 6)

8. (a) Show that,
   (i) $x \frac{d J_n(x)}{dx} = n J_n(x) - x J_{n+1}(x)$
   (ii) $n P_n = (2n - 1)x P_{n-1} - (n - 1) P_{n-2}$
   (b) Prove the recurrence relations for Laguerre's $[L_n(x)]$ polynomials of n[th] order
   $$xL'_n(x) = nL_n(x) - nL_{n-1}(x)$$

(6, 6)

UNIT-V

9. (a) State Cayley-Hamilton theorem.
   (b) Define ordinary and regular singular points.
   (c) Plot the variation of Bessel function, $J_0(x)$ and $J_2(x)$ with x.
   (d) Show that, $\Gamma(n + 1) = n\,\Gamma(n)$
   (e) What are the different types of singularities?
   (f) Define divergence of a vector function and give its physical significance.

(2 x 6 = 12)

x-x-x