

2071
B.A./B.Sc. (General) Second Semester
Mathematics
Paper – I: Solid Geometry

Time allowed: 3 Hours

Max. Marks: 30

NOTE: Attempt five questions in all, selecting atleast two questions each Unit.

x-x-x

UNIT – I

- I. a) Shift the origin to a suitable point so that the equation $2x^2 - 3y^2 + z^2 + 8x + 6y + 3 = 0$ is transformed into an equation in which the first degree terms are absent.
- b) Transform the equation $x^2 + 7y^2 + z^2 + 8yz + 16zx - 8xy - 9 = 0$ referred to axis as the lines joining the origin to the points (1,2,2), (2,-2,1) and (2,1,-2) (2x3)
- II. a) A sphere of constant radius k passes through the origin and meets the axis in A, B, C. Prove that the centroid of the triangle ABC lies on the sphere $9(x^2 + y^2 + z^2) = 4k^2$.
- b) Find the equation of the sphere which passes through the circle $x^2 + y^2 = 4, z = 0$ and is cut by the plane $x + 2y + 2z = 0$ in a circle of radius 3. (2x3)
- III. a) Prove that the spheres $x^2 + y^2 + z^2 + 2ax + c = 0$ and $x^2 + y^2 + z^2 + 2by + c = 0$ touch iff $\frac{1}{a^2} + \frac{1}{b^2} = \frac{1}{c}$, $a^2, b^2 > c > 0$.
- b) Find the limiting points of the co-axial system of sphere. $x^2 + y^2 + z^2 - 20x + 30y - 40z + 29 + \lambda(2x - 3y + 4z) = 0$. (2x3)
- IV. a) Find the equation of right circular cylinder whose generating circle is $x^2 + y^2 + z^2 = 4, x + y + z = 3$

P.T.O.

(2)

- b) Find the equation of right circular cylinder which envelops a sphere of centre (a,b,c) and radius r and has its generators parallel to the direction $\langle l, m, n \rangle$ (2x3)

UNIT – II

- V. a) Find the equation of the cone with vertex at the origin and which passes through the curve given by

$$x^2 + y^2 + z^2 - x - 1 = 0, x^2 + y^2 + z^2 + y - 2 = 0.$$

- b) Find the equation of the cone of revolution with vertex at the origin, the axis as the y-axis and semi-vertical angle 30° . (2x3)

- VI. a) Find the equation of the cone passing through coordinate axes and the three

mutually perpendicular lines $\frac{x}{2} = y = -z, x = \frac{y}{3} = \frac{z}{5}, \frac{x}{8} = \frac{y}{-11} = \frac{z}{5}.$

- b) Find the equation of the cone whose vertex is at the point (-1,1,2) and whose guiding curve is $3x^2 - y^2 = 1, z = 0$. (2x3)

- VII. a) Prove that the lines drawn from the origin so as to touch the sphere

$$x^2 + y^2 + z^2 + 2ux + 2vy + 2wz + d = 0 \text{ lie on the cone}$$

$$d(x^2 + y^2 + z^2) = (ux + vy + wz)^2.$$

- b) Show that $2y^2 - 8yz - 4zx - 8xy + 6x - 4y - 2z + 5 = 0$ represents a cone. Find the coordinates of its vertex. (2x3)

- VIII. a) identify the surface

$$x^2 + 4y^2 + 3z^2 + 2x - 8y + 9z - 10 = 0$$

- b) Show that the curve

$$6x^2 - 3y^2 - 2z^2 + 12x + 12y - 12 = 0 \text{ represents hyperboloid of two sheets.}$$

(2x3)