Exam Code: 0004
Sub. Code: 0341

2071

B.A./B.Sc. (General) Fourth Semester

Mathematics

Paper – I: Advanced Calculus - II

Time allowed: 3 Hours | Max. Marks: 30

NOTE: *Attempt five questions in all, selecting atleast two questions each Unit. All questions carry equal marks.*

x-x-x

UNIT – I

1. (a) Show that the sequence $\{-2^n\}$ diverges to $-\infty$.

(b) Show that $\lim_{n\to\infty}\left[\frac{1}{n^2}+\frac{1}{(n+1)^2}+\frac{1}{(n+2)^2}+....+\frac{1}{(2n)^2}\right]=0.$

2. (a) Let $\{x_n\}$ be a sequence defined as $x_1=\frac{3}{2},\ x_{n+1}=2-\frac{1}{x_n}\ \forall\ n\in N.$

Show that $\{x_n\}$ is monotonic and bounded. Also find limit of the sequence.

(b) If $x_n>0\ \forall\ n$ and $\lim_{n\to\infty}x_n=l$, then show that $\lim_{n\to\infty}\sqrt[n]{x_1x_2x_3\ ......x_n}=l.$

3. (a) Using Cauchy's general principle of convergence, show that sequence $\{a_n\}$ converges,

where $a_n=1+\frac{1}{2^2}+\frac{1}{3^2}+......+\frac{1}{n^2}$.

(b) Show that the sequence $\{a_n\}$ converges, where

$$a_n=1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+....+\frac{(-1)^{n-1}}{n}.$$

4. (a) Using concept of sequential continuity, show that the function

$$f(x)=\begin{cases}1, & \text{if } x \text{ is rational}\\ -1, & \text{if } x \text{ is irrational}\end{cases}$$

is discontinuous at every point of **R**.

(b) Show that $f(x)=x^2$ is uniformly continuous on $(-1, 1]$.

P.T.O.

UNIT - II

5. (*a*) Show that the series $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} \frac{a_n}{1+a_n}$, $a_n > 0 \ \forall \ n$ converges or diverges together.

(*b*) Show that the series $\sum_{n=1}^{\infty} \frac{1}{n^p}$, $p > 1$ converges and its sum lies between $\frac{1}{p-1}$ and $\frac{p}{p-1}$.

6. (*a*) Examine the convergence or divergence of the series $\sum_{n=1}^{\infty} \frac{(n-\log n)^n}{2^n n^n}$.

(*b*) Examine the convergence or divergence of the series $\sum_{n=1}^{\infty} \left[\frac{1.3.5 \ldots\ldots (2n-1)}{2.4.6 \ldots\ldots (2n)}\right]^p$.

7. (*a*) Show that the following series is convergent

$$\frac{1}{2^3} - \frac{1+2}{3^3} + \frac{1+2+3}{4^3} - \frac{1+2+3+4}{5^3} + \ldots\ldots .$$

(*b*) Discuss the convergence or divergence of the series

$$1 + \frac{2x}{2!} + \frac{3^2x^2}{3!} + \frac{4^3x^3}{4!} + \ldots\ldots, x > 0.$$

8. (*a*) Show that the following series is convergent :

$$\frac{1}{2^3} - \frac{1+2}{3^3} + \frac{1+2+3}{4^3} - \frac{1+2+3+4}{5^3} + \ldots\ldots .$$

(*b*) Find how the series $1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \ldots\ldots$ be arranged so that the sum is doubled.

x-x-x