

2071
B.A./B.Sc. (General) Fourth Semester
Mathematics
Paper – II: Differential Equation - II

Time allowed: 3 Hours

Max. Marks: 30

NOTE: Attempt five questions in all, selecting atleast two questions each Unit.

x-x-x

UNIT – I

- I. Find the solution in series $x \frac{d^2 y}{dx^2} + \frac{dy}{dx} + xy = 0$ (6)
- II. a) Show that $J_{\frac{1}{2}}(x) = \sqrt{\frac{2}{\pi x}} \sin x$
- b) Prove that $\int_0^{\infty} e^{-ax} J_0(bx) dx = \frac{1}{\sqrt{a^2 + b^2}} a > 0$ (2x3)
- III. a) Prove that $(1 - 2xh + h^2)^{-\frac{1}{2}} = \sum_{n=0}^{\infty} h^n P_n(x)$
- b) Prove that $\int_{-1}^1 P_n(x) P_m(x) dx = 0$ if $m \neq n$. (2x3)
- IV. a) If $f(x + y + z, x^2 + y^2 - z^2) = 0$ form the partial differential equation by eliminating function f .
- b) Find the general solution of $y z p + z x q = xy$. (2x3)

UNIT – II

- V. a) Prove that $\int_0^{\infty} \cos x^2 dx = \sqrt{\frac{\pi}{8}}$
- b) Evaluate $\int_0^{\infty} t e^{-t} \cos t dt$ (2x3)

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(2)

VI. a) Evaluate $L^{-1}\left(\frac{1}{s}\log\left(\frac{s+2}{s+1}\right)\right)$

b) State and prove convolution theorem. (2x3)

VII. a) State and prove second shifting property of Laplace transforms.

b) Evaluate $L^{-1}\left(\frac{se^{-\pi/2}s}{s^2+1}\right)$ (2x3)

VIII. a) Solve $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = 64\sin 2t$
 $y(0) = 0, y'(0) = 1$

b) Solve $\frac{dx}{dt} = ax + by$

$$\frac{dy}{dt} = bx + ay$$

given $x(0) = a$ and $y(0) = b$ (2x3)

x-x-x