Exam Code: 0006
Sub. Code: 0541

2071
B.A./B.Sc. (General) Sixth Semester
Mathematics
Paper – I: Analysis - II

Time allowed: 3 Hours | Max. Marks: 30

NOTE: *Attempt five questions in all, selecting atleast two questions each Unit.*

x-x-x

UNIT – I

I. a) Consider the region S = $\{(a,b) : 1\leq a \leq 2, 3 \leq b\leq 4)\}$

Let $f : S \to IR$ be defined by

$$f(a,b) = \begin{cases} 2 \text{ if } a \text{ is rational} \\ -2 \text{ if } a \text{ is irrational} \end{cases}$$

Show that f is not integrable over the region S.

b) Evaluate $\iint_S x^2 dxdy$ where S is the region enclosed by four parabolas $y^2 = x$, $y^2 = 4x$, $x^2 = 8y$, $x^2 = 16y$. (2x3)

II. a) Find the volume of a truncated cone with end radii 2 and 6 and height 8 units.

b) Change the order of integration of $\int_0^{2a}\int_{\sqrt{2ax-x^2}}^{\sqrt{2ax}} f(x,y)dydx$. Hence evaluate for $f(x,y) = 1$. (2x3)

III. a) State and prove Gauss's divergence theorem.

b) Verify Green's theorem in the plane for $\oint_C[(3x^2-8y^2)dx+(4y-6xy)dy]$ where C is the boundary of the region bounded by $y = \sqrt{x}$ and $y = x^2$.

IV. a) Apply Stoke's theorem to evaluate $\oint_C(ydx+zdy+xdz)$ where C is curve of intersection of the sphere $x^2 + y^2 + z^2 = a^2$ and the plane $x + z = a$.

P.T.O.

b) Use Gauss Divergence theorem to evaluate $\int_s \vec{f}.\vec{ds}$ where $\vec{f} = x^3\hat{i} + y^3\hat{j} + z^3\hat{k}$ and S is the surface of the sphere $x^2 + y^2 + z^2 = a^2$. (2x3)

UNIT – II

V. a) Prove that a sequence of functions $\{l_n(x)\}$ defined on a set S. Converges uniformly on S iff for every $\varepsilon > 0$ and for all x ε s there exists a positive integer N such that $| I_{n+p} (x) - I_n(x)| < \varepsilon \ \forall n \geq N, p \geq 1$

b) Apply Weirstrass-M-Test to show that the series $\sum_{n=1}^{\infty} \frac{a_n x^n}{1+x^{2n}}$ converges uniformly $\forall$ x ε *IR* if $\sum_{n=1}^{\infty} a_n$ is absolutely convergent. (2x3)

VI. a) Shat that the series $\sum_{k=1}^{\infty} \left[\frac{kx}{1+k^2x^2} - \frac{(k-1)x}{1+(k-1)^2x^2} \right]$ can be integrated term by term on [0,1] although it is not uniformly convergent in [0,1]

b) Let $\{f_n(x)\}$ be a sequence of real valued function defined on the interval [a,b] and bounded on [a,b] and let fb ∈ R[a,b] for n = 1,2,3, …….. If $\{f_n(x)\}$ converges uniformly to the function f on [a,b] then prove that f ∈ R [a,b] where R [a,b] denote set of functions which are Reimann Integrable on [a,b]. (2x3)

VII. a) When $-\pi < x < \pi$ prove that $\sin mx = \frac{2\sin m\pi}{\pi} \left[\frac{\sin x}{1^2 - m^2} + \frac{2\sin 2x}{2^2 - m^2} + \frac{3\sin 3x}{3^2 - m^2} + ... \right]$

b) Obtain Fourier Series for the function f(x) given by

$$f(x) = \begin{cases} 1 + \frac{2x}{\pi} & for \quad -\pi < x < 0 \\ 1 - \frac{2x}{\pi} & for \quad 0 < x < \pi \end{cases}$$

. Hence deduce that $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + ... = \frac{\pi^2}{8}$.

(2x3)

VIII. Show that :-

a) $\tan^{-1} x = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \ldots\ldots\ldots\ldots for -1 \le x \le 1$

b) $\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \ldots\ldots\ldots$

c) $\frac{1}{2}\left(\tan^{-1} x\right)^2 = \frac{x^2}{2} - \frac{x^4}{4}\left(1 + \frac{1}{3}\right) + \frac{x^6}{6}\left(1 + \frac{1}{3} + \frac{1}{5}\right) + \ldots\ldots\ldots where -1 < x \le 1$ (6)

x-x-x