Exam Code: 0006
Sub. Code: 0542

2071

B.A./B.Sc. (General) Sixth Semester

Mathematics

Paper – II: Linear Algebra

Time allowed: 3 Hours

Max. Marks: 30

NOTE: *Attempt five questions in all, selecting atleast two questions each Unit. All questions carry equal marks.*

x-x-x

UNIT – I

Q.No:1 (a) Does the set of all lower triangular matrices of order n over $\mathbb{R}$ form a vector space over $\mathbb{R}$ or not with respect to usual addition and scalar multiplication of matrices? Justify.

(b) Give an example to show that the union of two subspaces of a vector space V over a field F is not always a subspace of V. Also state a condition under which the union of two subspaces can be a subspace and prove it.

Q.No:2 (a) Let V be the vector space of polynomials of degree ≤ 3 over $\mathbb{R}$. Discuss if the vectors $v_1 = t^3 - 3t^2 + 5t + 1$, $v_2 = t^3 - t^2 + t + 1$, $v_3 = 5t^3 + 8t^2 + t + 3$ are linear independent or linearly dependent?

(b) Let V be a vector space over the field F. Prove that the set S of non-zero vectors $v_1, v_2, ..., v_n \in V$ is linearly dependent iff some vector, say v_k, can be expressed as the linear combination of the other vectors of the set S.

Q.No:3 (a) Let $W(F)$ be a subspace of a finite dimensional vector space $V(F)$. Prove that dim W$\leq$ dim V. Also prove that $V = W$ if and only if dimV=dim W.

(b) Find a basis and dimension of the solution space S of the following linear equations

$$\begin{aligned} x + 2y - 2z + t &= 0, \\ 2x + 4y - 2z + 4t &= 0, \\ 2x + 4y - 6z &= 0, \\ 3x + 6y - 8z + t &= 0. \end{aligned}$$

Q.No:4 (a) Find a linear map $T : \mathbb{R}^2 \to \mathbb{R}^3$ such that $T(1, 2) = (3, -1, 5)$ and $T(1, 0) = (2, 1, 1)$. Also determine null space, range space, nullity and rank of T.

(b) State and prove Rank Nullity Theorem.

P.T.O.

UNIT - II

Q.No:5 **(a)** Let $B = \{v_1, v_2, ..., v_n\}$ be basis of a vector space $V(F)$ and T be a linear transformation on V. Then prove that for any vector $v \in V$, $[T; B][v; B] = [T(v); B]$.

(b) Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ be a linear operator defined by $T(x, y, z) = (3x - z, 2x + y, -x + 2y + 4z)$. Prove that T is invertible and find explicit formula for T^{-1}.

Q.No:6 **(a)** Let a linear operator $T : \mathbb{R}^2 \to \mathbb{R}^2$ be defined by $T(x, y) = (3x + y, 3x + 5y)$. Find all the eigen values and basis for each eigen space.

(b) Let T be a linear operator on a finite dimensional vector space V(F). Prove that the following statements are equivalent:
(i) λ is an eigen value of T
(ii) The operator $T - \lambda I$ is singular
(iii) $\text{Det}(T - \lambda I) = 0$.

Q.No:7 **(a)** Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ be defined by $T(x, y, z) = (x - 3y + 3z, 3x - 5y + 3z, 6x - 6y + 4z)$. Is T diagonalizable? Justify. If T is diagonalizable find an invertible matrix P such that $P^{-1}AP$ is a diagonal matrix where A is the matrix of T with respect to usual basis B of $\mathbb{R}^3$.

(b) Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be defined by $T(x, y) = (x + y, x - y)$. Find the characteristic polynomial of T and verify Cayley-Hamilton Theorem.

Q.No:8 **(a)** Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ be defined by $T(x, y, z) = (2x - y, x + y + z, 2z)$. Find the characteristic and minimal polynomial of T.

(b) Prove that the characteristic and minimal polynomial of a linear operator have same irreducible factors.

x-x-x