Exam Code: 0001
Sub. Code: 0043

2012
B.A./B.Sc. (General) First Semester
Mathematics
Paper – I: Plane Geometry

Time allowed: 3 Hours | Max. Marks: 30

NOTE: *Attempt five questions in all, selecting atleast two questions each Unit.*

x-x-x

UNIT – I

I. a) Through what angle, the exes should be rotated to remove the mixed term in the equation $5x^2 - 4xy + 5y^2 - 3x + 4y - 5 = 0$. Also find the transformed equation.

b) Show that if $ax^2 + 2hxy + by^2 = 1$ and $a'x^2 + 2h'xy + b'y^2 = 1$ represents the same conic and exes are rectangular, then $(a-b)^2 + 4h^2 = (a'-b')^2 + 4h'^2$ (2x3)

II. a) Prove that the equation $ax^2 + 2hxy + by^2 + 2gx + 2fy$ represents two parallel lines if $h^2 = ab$ and $bg^2 = af^2$. Also find the distance between them.

b) Show that the equation $x^2 - 2xy\sec\theta + y^2 = 0$ represent a pair of straight line through the origin. Also find the separate equations $(\theta \neq 0)$. (2x3)

III. a) Find the equation of circle which touches y-axis at a distance 2 units from the origin and cuts an intercepts of 3 units from positive x-axis.

b) Find the locus of the point of interaction of perpendicular tangents to the circle $x^2 + y^2 = r^2$. (2x3)

IV. a) Prove that the two circles $x^2 + y^2 + 2ax + c = 0$, $x^2 + y^2 + 2by + C = 0$ touch if

$$\frac{1}{a^2} + \frac{1}{b^2} = \frac{1}{c}$$

b) If the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ is a circle of a co-axial system having the origin as its limiting points, also find the other limiting points. (2x3)

P.T.O.

UNIT – II

V. a) Show that normals at the extremities of the latus rectum of a parabola $y^2 = 4ax$ intersect at right angles on the axis of the parabola.

b) Find the joint equation of pair of tangents to the parabola $y^2 = 8x$ from the point (1,3). Find the angle between them. (2x3)

VI. a) Tangents at the ends of a focal chord of a parabola intersect at right angle on directrix.

b) Prove that the product of focal distances of an extremity of a semi diameter of an ellipse is equal to the square of the conjugate semi diameter. (2x3)

VII. a) Prove that the condition that the pole $lx + my = 1$ w.r.t the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ may lie on ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 9$ is $a^2l^2 - b^2m^2 = 9$.

b) Find the equation of the asymptotes to hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$. (2x3)

VIII. a) Prove that, of the two conjugate diameters of a hyperbola $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, only one of them intersects the hyperbola, while other interests the conjugate hyperbola in two distinct real points.

b) Reduce the equation $x^2 - 3xy + y^2 + 10x - 10y + 21 = 0$ to standard form and find the equation of axes. (2x3)

x-x-x