Exam Code: 0001
Sub. Code: 0045

2012
B.A./B.Sc. (General) First Semester
Mathematics
Paper – III: Trigonometry and Matrices

Time allowed: 3 Hours Max. Marks: 30

NOTE: *Attempt five questions in all, selecting atleast two questions each Unit.*

x-x-x

UNIT – I

I. a) If α, β are roots of $x^2 - 2x + 2 = 0$, then prove that $\frac{(z+\alpha)^n - (z+\beta)^n}{\alpha - \beta} = \frac{\sin n\phi}{\sin^n \phi}$ where $z + 1 = \cot\phi$

b) If $x = cis\alpha$, $\sqrt{1-c^2} = nc - 1$, then prove $1 + c\cos\alpha = \frac{c}{2n}(1+nx)\left(1+\frac{n}{x}\right)$ (2x3)

II. a) Find the product $(x-\xi)(x-\xi^3)(x-\xi^5)(x-\xi^7)$, ξ is a primitive 8^{th} root of unity.

b) Find all the values of $\left(1+\sqrt{3}i\right)^{3/4}$ and find their continued product. (2x3)

III. a) Expand $\sin^7\theta$ in a series of sines of multiple of θ.

b) If $\frac{(1+i)^{p+iq}}{(1-i)^{p-iq}} = \alpha + i\beta$ then prove that $\tan^{-1}\frac{\beta}{\alpha} = \frac{p\pi}{2} + q\log 2$, when only principal values are considered. (2x3)

IV. a) Using Gregory series, prove that $1 + \frac{1}{3} - \frac{1}{5} - \frac{1}{7} + ... = \frac{\pi}{2\sqrt{2}}$

b) Sum the series $1 + x\cos\theta + \frac{x^2\cos 2\theta}{\lfloor 2} + \frac{x^3\cos 3\theta}{\lfloor 3} + ...\infty$ (2x3)

UNIT – II

V. a) Express matrix $\begin{bmatrix} 2-i & 3 & 1+i \\ -5 & 0 & -6i \\ 7 & i & -3+2i \end{bmatrix}$ as a sum a of Hermitian and a skew Hermitian matrix.

P.T.O.

b) If A is a non-zero column matrix. B is a non-zero row matrix then show $\rho(AB) = 1$. (2x3)

VI. a) Find rank of the matrix $A = \begin{bmatrix} 9 & 0 & 2 & 3 \\ 0 & 1 & 5 & 6 \\ 4 & 5 & 3 & 0 \end{bmatrix}$ by reducing it to normal form.

b) Find value of k for which the equations, $x + ky + 3z = 0$, $4x + 3y + kz = 0$, $2x + y + 2z = 0$ have a non-trivial solution. (2x3)

VII. a) For what value of λ the following system of equations has a solution? Find also the solution in each case $x + y + z = 1$, $x + 2y + 4z = \lambda$, $x + 4y + 10z = \lambda^2$.

b) If λ is an eigen value of a non-singular matrix A, then prove $\frac{|A|}{\lambda}$ is an eigen value of adj A. (2x3)

VIII. a) Verify Cayley-Hamilton theorem for Matrix A, $A = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & -1 \\ 3 & -1 & 1 \end{bmatrix}$. Hence find A^{-1}.

b) Find eigen values and the corresponding eigen vectors of matrix $A = \begin{bmatrix} 7 & 4 & -1 \\ 4 & 7 & -1 \\ -4 & -4 & 4 \end{bmatrix}$ Is A diagonalizable? (2x3)

x-x-x