Exam.Code:0005
Sub. Code: 0443

2012
B.A./B.Sc. (General), Fifth Semester
Mathematics
Paper – I: Analysis - I

Time allowed: 3 Hours Max. Marks: 30

***NOTE**: Attempt five questions in all, selecting atleast two questions from each Unit.*

x-x-x

UNIT – I

I. a) Show that the set of irrational numbers is uncountable.

b) Let f(x) x^4 where $P = \{-2, -\frac{1}{3}, 2, 4\}$ find L (P, f) and U (P, f). (3,3)

II. a) If f is continuous on [a,b] then prove that f is Riemann integrable on [a,b].

b) Prove that $\frac{\pi}{6} \leq \int_0^{\frac{1}{2}} \frac{dt}{\sqrt{(1-t^2)(1-k^2t^2)}} \leq \frac{\pi}{3\sqrt{4-k^2}}$ where $k^2 < 4$. (3,3)

III. a) State and prove fundamentals theorem on integral calculus.

b) If f is bounded and integrable in (a,b], then prove that |f| is also integrable on (a,b]. Moreover $\left|\int_a^b f dx\right| \leq \int_a^b |f| dx$. Prove it. (3,3)

IV. a) Show that $\int_0^1 \frac{x^{m-1}(1-x)^{n-1}}{(a+bx)^{m+n}} dx = \frac{1}{(a+b)^m a^n} B(m,n)$

b) State and prove duplication formula of Legendre by using Beta and Gamma functions. (3,3)

UNIT – II

V. a) Show that $\int_1^\infty \left(\frac{1}{x} - \frac{1}{\sinh x}\right) \frac{dx}{x}$ is convergent.

b) Discuss the convergence of $\int_0^1 \left(\log \frac{1}{x}\right)^m dx$ (3,3)

P.T.O.

VI. a) Using Dirichlet's test discuss the convergence of $\int_0^\infty e^{-ax^2} \cos bx \, dx$ where a > 0.

b) Show that $\int_\pi^\infty \frac{\sin x}{x} dx$ is conditionally convergent. (3,3)

VII. a) Show that $\int_0^{\frac{\pi}{2}} \log\left(\frac{a+b\sin\theta}{a-b\sin\theta}\right) \text{cosec}\theta \, d\theta = \pi \sin^{-1}\frac{b}{a}$

b) Show that $\int_0^\infty \frac{b\log(1+ax)-a\log(a+bx)}{x^2} dx = ab\log\frac{b}{a}$ where p, q, a, b > 0. (3,3)

VIII. a) Prove that $\int_0^{\frac{\pi}{2}} \frac{1}{\left(a^2\sin^2 x + b^2\cos^2 x\right)^2} dx = \frac{\pi\left(a^2+b^2\right)}{4a^3b^3}$

b) Discuss the convergence of $\int_0^1 \frac{\left(x^m + x^{-m}\right)\log(1+x)}{x} dx$ (3,3)

x-x-x