Exam.Code:0005
Sub. Code: 0444

2012

B.A./B.Sc. (General), Fifth Semester

Mathematics

Paper – II: Modern Algebra

Time allowed: 3 Hours | Max. Marks: 30

***NOTE**: Attempt five questions in all, selecting atleast two questions from each Unit.*

x-x-x

UNIT- I

1. (a) Give an example of an abelian group of order 8, clearly proving all the axioms.
   (b) Let G be a semi-group and $a, b \in G$. Prove that G is a group if and only if both the equations $ax = b$ and $ya = b$ have unique solutions in G. (2, 4)

2. (a) Let G be a finite group and $a \in G$. Prove that $a^t = e$ if and only if $O(a)$ divides t.
   (b) If H and K are two subgroups of a group G, then prove that HK is a subgroup iff $HK = KH$. (3, 3)

3. (a) State and prove Lagrange's theorem. Is converse true? Justify.
   (b) Prove that a subgroup of a cyclic group is again cyclic. (3, 3)

4. (a) Prove that a subgroup H of a group G is a normal subgroup of G iff the product of two right cosets of H in G is again a right coset of H in G.
   (b) If H is a normal subgroup of A_n and contains a cycle of length 3, then prove that $H = A_n$ $(n \geq 5)$. (3, 3)

UNIT- II

5. (a) Prove that the set $R = \{a + \sqrt{2}\ b : a, b \in Q\}$ where Q is the set of rational, is a ring.
   (b) Let I and J be any two ideals of a ring R. Prove that $I \cup J$ is an ideal of R iff either $I \subseteq J$ or $J \subseteq I$. (3, 3)

6. (a) Prove that an ideal S of the ring Z of all integers is a maximal ideal if and only if S is generated by some prime integer.
   (b) Write down all the distinct elements of the ring $Z/4Z$. Write the multiplication table for this ring. (3, 3)

P.T.O.

7. (a) Let I and J be any two ideals of a ring R. Then prove that $I/(I \cap J) \cong (I + J)/J$.

(b) Find all homomorphisms from the ring $\mathbf{Z}$ onto $\mathbf{Z}$. (3, 3)

8. (a) Let R be a commutative ring with identity, $f(x) \in R[x]$ and $a \in R$. Then show that $(x - a)$ divides $f(x)$ if and only if a is a root of $f(x)$.

(b) Let R be a commutative ring with unity and $< x >$ is a prime ideal of $R[x]$. Show that R is an integral domain. (3, 3)

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