Exam Code: 0001
Sub. Code: 0044

2012
B.A./B.Sc. (General) First Semester
Mathematics
Paper – II: Calculus - I

Time allowed: 3 Hours Max. Marks: 30

NOTE: *Attempt five questions in all, selecting atleast two questions each Unit.*

x-x-x

UNIT – I

I. a) If $a > 0$ and $b \in R$, then prove that there exists a natural no. n such that $na > b$ and hence prove that set of natural numbers N is not bounded above.

b) Find the least upper bound and greatest lower bound of the set $\left\{\frac{2x-1}{x+4} : |x-5|<2\right\}$

(2x3)

II. a) Solve the inequality $\left|\frac{x-3}{x+5}\right| > 2$

b) Prove that $\lim_{x\to 0} \sin\frac{1}{x}$ does not exist. (2x3)

III. a) Let f be a continuous function on [a,b], then prove that f attains its l.u.b. and g.l.b. in [a,b]

b) If the function f(x) defined by $f(x)=\begin{cases} 3ax+b & if \quad x>1 \\ 11 & if \quad x=1 \\ 5ax-2b & if \quad x<1 \end{cases}$ is continuous at x =1.

Find the values of a and b. (2x3)

IV. a) Evaluate $\lim_{x\to 0}\left(\cot^2 x-\frac{1}{x^2}\right)$

b) Find the values of a,b,c if $\lim_{x\to 0}\frac{ae^x-b\cos x+ce^{-x}}{x\sin x}=2$ (2x3)

P.T.O.

UNIT – II

V. a) State and prove Rolle 's Theorem.

b) Use mean value theorem to prove that $\frac{x}{1+x^2} \le \tan^{-1} x \le x$ for $x \ge 0$ (2x3)

VI. a) Use Taylor's theorem to expand f(x) = $e^{\cos x}$ in powers of $x - \frac{\pi}{2}$

b) Differentiate $y = x^{\sinh x} + x^{\cosh x}$ w.r.t. x. (2x3)

VII. a) Prove that the function $2\tanh^{-1}\left(\tan\frac{x}{2}\right)$ and $\cosh^{-1}(\sec x)$ can differ only by a constant.

b) Using Maclaurin's theorem, expand the function $\sin^{-1} x$. (2x3)

VIII. a) If y = log (1 + cosx), prove that $y_1 y_2 + y_3 = 0$.

b) If y = sin (m $\sin^{-1}$x), show that $(1 - x^2)y_{n+2} = (2n+1)x\, y_{n+1} + (n^2 - m^2)y_n$. Also find $y_n(0)$. (2x3)

x-x-x