Exam.Code:0005
Sub. Code: 0445

2012

B.A./B.Sc. (General), Fifth Semester

Mathematics

Paper – III: Probability Theory

Time allowed: 3 Hours Max. Marks: 30

NOTE: *Attempt five questions in all, selecting atleast two questions from each Unit.*

x-x-x

UNIT – I

I. a) For any n events $A_1, A_2, \ldots\ldots, A_n$, prove that $P\left(\bigcap_{i=1}^{n} Ai\right) \geq \sum_{i=1}^{n} P(Ai) - (n-1)$.

b) The probabilities of x, y and z becoming managers are $\frac{4}{9}, \frac{2}{9}$ and $\frac{1}{3}$ respectively.

The probabilities that the bonus scheme will be introduced if x, y and z becomes managers are $\frac{3}{10}, \frac{1}{2}$ and $\frac{4}{5}$ respectively.

i) What is the probability that bonus scheme will be introduced?

ii) If the bonus scheme has been introduced, what is the probability that the manager appointed was x? (3,3)

II. a) Let x be a random variable with probability mass function given by

$$P(X = x) = \begin{cases} \frac{x}{10}; & x = 0,1,2,3,4 \\ 0; & elsewhere \end{cases}. \text{ Find } P\left(\frac{1}{3} < x < \frac{7}{2} \middle| x > 1\right).$$

b) Let x be a random variable having p.d.f. $f(x) = \begin{cases} 6x(1-x), & 0 < x < 1 \\ 0, & elsewhere \end{cases}$. Then find

i) E (x)

ii) Variance of x

iii) $P(\mu - 2\sigma < x < \mu + 2\sigma)$ (3,3)

P.T.O.

III. a) Let x be a continuous random variable having p.d.f. $f(x)=\begin{Bmatrix} \frac{3}{4}x(2-x), & 0 \le x < 2 \\ 0 & , elsewhere \end{Bmatrix}$. Find measure of skewness and kurtosis of the distribution.

b) If a random variable x follows binomial distribution with parameters n and p, prove that $P(X = even) = \frac{1}{2}\left[1+(q-p)^n\right]$. (3,3)

IV. a) If x is a Poisson random variable with parameters m and u_r is the r^{th} central moment, then show that $\mu_{r+1} = m\left(r_{c_1}\mu_{r-1} + r_{c_2}\mu_{r-2} + \ldots\ldots + r_{c_r}\mu_0\right)$

b) If a fair coin is tossed 10 times, find the probability of getting

i) exactly six heads

ii) atleast six heads

iii) atmost six heads (3,3)

UNIT – II

V. a) Let x be uniformly distributed random variable in [-2,2]. Find $P\left(|x-1| \ge \frac{1}{2}\right)$

b) Prove that mean deviation about mean of an exponential distribution with parameter λ is $\frac{2}{\lambda}e^{-1}$. (3,3)

VI. a) Prove that in a normal distribution, mean, median and mode coincide.

b) Prove that $f(x) = \frac{1}{\sqrt{2\pi}b}e^{\frac{-1}{2}\left(\frac{x-a}{b}\right)^2}$, $-\infty < x < \infty$, b > 0 is a p.d.f. (3,3)

VII. a) Let X and Y be two random variables having joint p.d.f. $f(x,y)=\begin{Bmatrix} k(6-x-y), & 0<x<2, \ 2<y<4 \\ 0 & , elsewhere \end{Bmatrix}$.

Find (i) value of k (ii) P (x < 1/ y < 3) (iii) P (x + y < 4)

b) If M (t_1, t_2) is the movement generating function of two independent random variables X and Y, then show that $M(t_1, t_2) = M(t_1, 0)\, M(0, t_2)$ (3,3)

VIII. a) Two positively correlated variables X_1 and X_2 have variances σ_1^2 and σ_2^2 respectively. Determine the value of constant a such that $x_1 + ax_2$ and $x_1 + \frac{\sigma_1}{\sigma_2} x_2$ are uncorrected.

b) Let X and Y have a bivariate normal distribution with respective parameters $\mu_x = 2.8$, $\mu_y = 110$, $\sigma_x^2 = 0.16$, $\sigma_y^2 = 100$ and $r = 0.6$.

Compute:-

i) $P(106 < y < 124)$

ii) $P(106 < y < 124 \mid x = 3.2)$ (3,3)

x-x-x