

2012

B.Sc. (Hons.) Biotechnology

First Semester

BIOT-103A-T: Mathematics

Time allowed: 3 Hours

Max. Marks: 67

NOTE: Attempt five questions in all, including Question No. 1 (Section-A) which is compulsory and selecting two questions each from Section B-C.

X-X-X

Section A

1. (a) Let $A = \{1, 2, 3, 4, 5, 6, 8, 10\}$, $B = \{2, 4, 5, 7, 9, 10\}$ be two given sets and $X = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11\}$ be the universal set. Find $(A \cup B)'$ and $A - B$. 2
- (b) Evaluate $\int_{-1}^1 x \cos x \, dx$. 2
- (c) Find the median of the following data: 2, 4, 6, 3, 8, 9, 11, 1 2
- (d) Form a differential equation by eliminating arbitrary constants a and b from the relation $y = ae^{-2x} + be^{2x}$ 3
- (e) Solve the differential equation $\frac{dy}{dx} = \frac{y+1}{x-10}$ 2
- (f) Solve $\frac{d^2y}{dx^2} - 4y = 0$ 2
- (g) Find the relation R , if R is a relation on the set N of Natural numbers defined by $R = \{(x, y) : 2x + 3y = 48, x \in N, y \in N\}$ 2

Section B

2. (a) Reduce the complex number $\left(\frac{1}{1-4i} - \frac{2}{1-i}\right)\left(\frac{3+4i}{5-i}\right)$ to the standard form. 4
- (b) If $z_1 = 3 - i$, $z_2 = -1 - i$, then find the value of $\left|\frac{z_1 - z_2 - 1}{z_1 + z_2 + 1}\right|$ 4
- (c) Find the modulus and principal argument of $1 + i\sqrt{3}$. 5
Also find the complex conjugate of $(2 - 5i)^3$.
3. (a) Solve $\sqrt{3}x^2 - \sqrt{2}x + 3\sqrt{3} = 0$ 4
- (b) Prove that $\sqrt{2}$ is an irrational number. 5
- (c) Evaluate $\lim_{x \rightarrow \frac{\pi}{2}} \frac{\tan 2x}{x - \frac{\pi}{2}}$ 4

(2)

4. (a) Evaluate $\lim_{x \rightarrow 1} \left(\frac{x-2}{x^2-x} - \frac{1}{x^3-3x^2+2x} \right)$ 4
- (b) Find the domain and range of the function $f(x) = \sqrt{x^2-5x+6}$. 5
- (c) Let $A = \{x : 1 < x \leq 15, x \text{ is a prime}\}$ and $B = \{x : x^2 - (3+\sqrt{3})x + 3\sqrt{3} \leq 0, x \in \mathbb{R}\}$ be the two given sets. Let Set of real numbers denoted by \mathbb{R} be the Universal Set. Find $A \cap B$, $A \cup B$, $A \cap B^c$. 4
5. (a) Draw the graph of the function $f(x) = |2-3x|$ for $x \in \mathbb{R}$ 4
- (b) Let $f, g : \mathbb{R} \rightarrow \mathbb{R}$ be defined, respectively by $f(x) = x-3$, $g(x) = 3-2x$. Find $f \circ g, f \circ f, g \circ g$ and $g \circ f$. 5
- (c) Find real numbers a and b if $(a+2i)(b-3i)$ is the complex conjugate of $-5-4i$ 4

Section C

6. (a) Differentiate $\frac{4\sqrt{x} + 5 \sin x}{3x + 7 \cos x}$ with respect to x 4
- (b) Differentiate $(\sin x)^x + \sin^{-1}(\sqrt{x})$ with respect to x 5
- (c) Find the local maximum and minimum values of the function given by

$$f(x) = 2x^4 - 24x^2 + 107$$
 4
7. (a) Evaluate $\int \frac{x^2+4}{x^2-5x+6} dx$ 4
- (b) Evaluate $\int \frac{x+1}{\sqrt{5x^2-2x}} dx$ 5
- (c) Find the intervals in which the function $f(x) = (x+2)^3(x+3)^2$ is strictly increasing or strictly decreasing. 4
8. (a) Find the area enclosed by the ellipse $2x^2 + 3y^2 = 6$ 4
- (b) Solve the linear programming Problem : Minimize $Z = -50x + 20y$ subject to the constraints
 $2x - y \geq -5$, $3x + y \geq 3$, $2x - 3y \leq 12$, $x \geq 0, y \geq 0$. 5

(c) Draw the histogram and frequency polygon for the following data: 4

Seed yield(gms)	3.5-4.5	4.5-5.5	5.5-6.5	6.5-7.5	7.5-8.5	8.5-9.5	9.5-10.5
Number of Plants	2	5	3	7	8	10	15

9. (a) Calculate the Mean, Median and Mode for the following data: 7

Class Interval	20-30	30-40	40-50	50-60	60-70	70-80	80-90
Frequency	4	6	10	5	8	3	1

(b) The monthly sales of two products A and B were recorded as follows : 6

Product A	57	68	45	38	59	78
Product B	56	72	86	39	76	75

Find out which of the two products shows greater fluctuation in sales.